CONTROLLABILITY ANALYSIS OF A CLASS OF FRACTIONAL DIFFERENTIAL INCLUSIONS/SYSTEMS IN BANACH SPACES

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SUMMARY

The main aim of the thesis is to study the basic properties of the controllability analysis of a class of fractional differential inclusions/systems in Banach spaces. In this thesis we have investigated existence of the mild solutions for an impulsive fractional differential inclusions in Banach spaces, controllability of fractional impulsive differential inclusions with sectorial operator in Banach spaces, controllability of the Cauchy problem for fractional differential equations with delay and controllability of a class of fractional impulsive differential inclusions with non-local conditions.

In chapter 3, we have first considered existence of the mild solutions for an impulsive fractional differential inclusions involving the Caputo derivative in Banach spaces and the results are obtained by using Leray Schauder's fixed point theorem.

We have consider the existence of the mild solutions for an impulsive fractional differential inclusions of the form:

$$^{c}D_t^q x(t) \in Ax(t) + F(t, x(t))$$

$$\Delta x(t_k) = I_k(x(t_k^-)); k = 1, 2, ..., m$$

$$x(0) = x_0;\ 0 < T < b$$

where $A : D(A) \subset X \to X$ is a closed, densely defined linear operator and infinitesimal generator of a strongly continuous semigroup $\{S(t)\}_{t \geq 0}$ on Banach space X. $^{c}D_t^q$ denotes the Caputo fractional derivative of order q, the state $x(.)$ takes values in Banach space X, $F : J \times X \to 2^X \backslash \{\phi\}$ is a nonempty, bounded, closed and convex multivalued map. Here, $0 = t_0 < t_1 < t_2 < .... < t_m < t_{m+1} = T$; $I_k \in C(X, X)$, $k = 1, 2, ..., m$ are bounded functions, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$; $x(t_k^+) = \lim\limits_{h \to 0} x(t_k + h)$ and $x(t_k^-) = \lim\limits_{h \to 0} x(t_k - h)$ represents the right and left limits of $x(t)$ at

$t = t_k$ respectively.

In [32] authors have claimed that most of the published papers dealing with impulsive differential equation of fractional orders are not mathematically correct and they have introduced a new class of impulsive fractional problems with several fractional orders.

Based on the above fact, we have proved the existence and uniqueness of mild solutions for an impulsive fractional differential inclusions involving the Caputo derivative in Banach spaces by the new concept introduced by [32] which is the main motivation for this work.

In Chapter 4, we have proved the controllability of an impulsive fractional differential inclusions involving the Caputo derivative using sectorial operator in Banach spaces. The results are obtained by using fractional calculation, operator semigroup and Leray-Schauder fixed point theorem. An example is given to illustrate the theory.

In this work, we concerned with the controllability for an impulsive fractional differential inclusions of the form:

$$^cD_t^q x(t) \in Ax(t) + F(t, x(t)) + Bu(t);\ t \in J := [0, b];\ t \neq t_k$$

$$\Delta x(t_k) = I_k(x(t_k^-)); k = 1, 2, ..., m$$

$$x(0) = x_0$$

where A is a sectorial operator of the type (M, θ, q, μ) on Banach space X.
$^cD_t^q$ denotes the Caputo fractional derivative of order $0 < q < 1$, the state $x(.)$ takes values in Banach space X, $F : J \times X \to 2^X \backslash \{\phi\}$ is a nonempty, bounded, closed and convex multivalued map. Here, $0 = t_0 < t_1 < t_2 < .... < t_m < t_{m+1} = T$; $I_k \in C(X, X)$, $k = 1, 2, ..., m$ are bounded functions, B is a bounded linear operator from X into X, the control $u \in L^1(J, X)$, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$; $x(t_k^+) = \lim_{h\to 0} x(t_k + h)$ and $x(t_k^-) = \lim_{h\to 0} x(t_k - h)$ represents the right and left limits of $x(t)$ at $t = t_k$ respectively.

As discussed in chapter 3, in this work we have proved controllability of an impulsive fractional differential inclusions involving the Caputo derivative in Banach spaces by introducing Sectorial operator.

In chapter 5, we have studied the controllability result of the Cauchy problem for fractional differential equation with delay in Banach spaces using the theory of an analytic semigroups. We have confine in the Kuratowski measure of non-compactness and fixed point theorem.

In this chapter, we have proved controllability of the Cauchy problem for fractional differential equations with delay of the form:

$$^{c}D_t^q x(t) = Ax(t) + F(t, x_t) + Bu(t);\ t \in J := [0, T];$$

$$x(t) = \phi(t); t \in J_0 := [-b, 0];$$

$$x_t(\theta) = x(t+\theta); \theta \in [-b, 0] \text{ and } \phi \in C([-b, 0], X).$$

where $b, T > 0$; $D^q; q \in (0, 1)$ is the Liouville- Caputo fractional derivative of order q. A is the infinitesimal generator of an analytic semigroup $L(.)$ uniformly bounded linear operator on X. $F : J \times J_0 \to X$ is a given function; the state function $x(t)$ takes values in $\Lambda = (J_0, X)$ and the control $u \in L^2(J, U)$ a Banach space of admissible control functions with U as a Banach space. Here, x_t represents the history of the state from $-\infty$ upto the present time t. We assume that the histories x_t belongs to some abstract phase space Λ.
Also, $x_t : J_0 \to X$ is defined by $x_t(\theta) = x(t+\theta); \theta \in J_0$ and $\phi \in \Lambda$ where X is a Banach space with norm $||.||$. B is a bounded linear operator from U to X.

In [77] authors have presented new existence theorems of mild solutions to Cauchy problem for some fractional differential equations with delay and obtained results using theory of analytic semigroups and compact semigroups, the Kuratowski measure of non-compactness, and fixed point theorems, with the help of some estimations. In this chapter, we have extended the work done by the authors [77] and proved Controllability of the Cauchy Problem for Fractional Differential Equations with Delay in Banach spaces.

In chapter 6, we have proved the controllability results for the class of impulsive fractional differential inclusions. Under the non-local conditions, the controllability of the system is established by applying multivalued analysis, fractional calculus combined with the Krasnoselskii's multi-valued fixed point theorem and the contraction mapping. We have proved controllability of a class of impulsive fractional differential inclusions with non-local condition of the form:

$$
\begin{aligned}
&{}^{C}D_t^q x(t) \in Ax(t) + G(t, x(t)) + Bu(t);\ t \in J := [0, b]; t \neq t_p \\
&\Delta x(t_p) = I_p(x(t_p^-)); p = 1, 2, ..., k; \\
&x(0) + g_1(x) = x_0, x'(0) + g_2(x) = x'_0;
\end{aligned}
$$

where $A : D(A) \subset E \to E$ is a sectorial operator of the type (M, θ, q, μ) on Banach space E. ${}^{C}D_t^q$ is the Caputo fractional derivative of order $1 < q < 2$, the state $x(.)$ takes values in Banach space E. $G : J \times E \to 2^E \setminus \{\phi\}$ is a nonempty continuous multi-valued map. The control function $u \in L^2(J, U)$, a Banach space of admissible control functions with U as a Banach space. B is a bounded linear operator from U to E. E is a Banach space with norm $\|.\|$. Also $0 = t_0 < t_1 < t_2 < ... < t_p < t_{p+1} = T$; $I_p \in C(J, E); p = 1, 2, ..., k$ are bounded, $\Delta x(t_p) = x(t_p^+) - x(t_p^-)$; $x(t_p^+) = \lim_{h \to 0} x(t_p + h)$ and $x(t_p^-) = \lim_{h \to 0} x(t_p - h)$ represent the right and left limits of $x(t)$ at $t = t_p$ respectively. The non local conditions $x(0) + g_1(x) = x_0$ and $x'(0) + g_2(x) = x'_0$; where $x_0, x'_0 \in E$ have many applications in different physical problems than the classical initial condition $x(0) = x_0$.

In [35] Chalishajar and Acharya proved the controllability result for neutral impulsive differential inclusions with nonlocal conditions by using the fixed point theorem for condensing multi-valued map due to Martelli. In [7] the authors has discussed the existence of solution for Riemann-Liouville fractional differential inclusions using Krasnoselskii's multi-valued fixed point theorem. In [59] the authors has proved the controllability of a class of fractional differential inclusions with non-local conditions using Krasnoselskii's fixed point theorem and solution operator theorem. In extension of the above work, we have proved the controllability of a class of impulsive fractional differential inclusions with non-local conditions by using Krasnoselskii's multi-valued fixed point theorem.