Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF AGRICULTURE

B.Tech. (Agriculture Engineering) Summer 2018 - 19 Examination

Semester: 2 **Date: 15/04/2019**

Subject Code: 20103153 **Time: 02:00pm To 04:00pm**

Subject Name: Engineering Mathematics II **Total Marks: 50**

Instructions

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 A) Fill in the blanks **(05)**

i) Cauchy-Euler equations are differential equations with _______ coefficients.

ii) Ordinary differential equations have ____________ independent variable.

iii) A function $z = \frac{2}{z^2+1}$ is not analytic at___________

iv) ______________method is used to find complex function when either real or imaginary part is given.

v) Differential equations of order four have ____________ arbitrary constants in its general solution.

vi) Simultaneous differential equations have__________ dependent variable.

vii) The value of Fourier coefficient b_n for $f(x) = x^2$ in $(-1,1)$ is ____________

viii) $|x|$ is an ________ function.

ix) $J_n(x)$ is a Bessel function of order _________.

x) Frobenious method is used to obtain power series near ________ point.

A) Multiple Choice Questions . **(10)**

i) Singular points for $(x^2+1)\, y'' + xy' - y = 0$ are

a) $\pm i$ b) ±1 c) 0 d)None of the above

ii) The complex function $f(z) = \frac{1}{z^2-1}$ is not analytic on

a) ± 1 b) $\pm i$ c) ± 2 d)None of the above

iii) The differential equation $M(x,y)dx + N(x,y)dy = 0$ is exact if and only if ,

a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ c) $\frac{\partial M}{\partial x} = -\frac{\partial N}{\partial y}$ d) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

iv) Which of the following equation is linear in$y$?

a)$\frac{dy}{dx} + xy^2 = sinx$ b)$\frac{dy}{dx} + y = sinx$ c)$\frac{dy}{dx} + xy = y^2$ d)$\frac{dy}{dx} + xy^2 = e^x$

v) $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = sinx$ is a differential equation with order _____ and degree ___.

a) 2,3 b) 3,2 c) 4,2 d) 2,4

vi) The integrating factor for a linear equation $\frac{dx}{dy} + p(y)x = q(y)$ is given by ,

a) $e^{-\int p(x)dx}$ b) $e^{\int p(x)dx}$ c)$e^{\int p(y)dy}$ d) $e^{-\int q(x)dx}$

vii) General solution of $(D^2+1)y = 0$ is

a) $y = c_1 cosx + c_2\, sinx$ b) $y = (c_1 + c_2 x)e^{-x}$

c) $y = c_1 cost + c_2\, sint$ d) $y = (c_1 + c_2 t)e^{-t}$

viii) Wronskian of general solution $y = c_1 cosx + c_2 sinx$ is ,
a)1 b)$-sinx$ c) $cosx$ d) 0

ix) Cauchy-Riemann equations are,

a. $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial x}$ & $\frac{\partial v}{\partial x} = -\frac{\partial v}{\partial y}$ b. $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$ & $\frac{\partial v}{\partial y} = -\frac{\partial v}{\partial x}$

c. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ d. none of the above

x) Function $f(z) = \frac{3}{z^2+2z+1}$ is discontinuous on point,

a) -1 b) 2 c) 0 d) 1

xi) Complex conjugate of $z = -2 + 6i$ is

a)$z = -2 - 6i$ b) $z = 2 - 6i$ c) $z = 2 + 6i$ d) $z = -2 + 6i$

xii) $|z|$ of $z = 2 + 3i$ is,

a)$\sqrt{13}$ b13 c) 4 d) $\sqrt{5}$

xiii) Order of $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} cosx$ is

a) 3 b)2 c) -1 d) 4

xiv) Partial differential equation $\frac{\partial^3 u}{\partial x \partial t \partial y} = e^{-t} cosx\, siny$ have___ independent variables,

a) 4 b)2 c) 1 d) 3

xv) f_{xx} of $f = 2x^2 + y^3$ is,

a) 2 b)$2x$ c) 4 d) $4x$

xvi) Laplace equation is ,

a) $f_{xx} + f_{yy} = 0$ b)$f_{xx} - f_{yy} = 0$ c) $f_{xy} + f_{yx} = 0$ d) $4f_{xx} + 5f_{xy} = 0$

xvii) Real part of $z = e^z$

a) $e^x cosy$ b) $e^x siny$ c) 0 d) e^x

xviii) $If\ z_1 = 2+3i$ and $z_2 = 3+3i$ then $z_1 + z_2$ is ,

a) 5+6i b) $6-6i$ c) $5-6i$ d) $-1+0i$

xix) Clairauts equation is of the form

a) $z = px + qy + f(p,q)$ b) $f(z,p,q) = 0$

c) $f(p,q) = 0$ d) $f(x,y,z) = 0$

xx) $\lim_{z\to -1} z^2 + 1$ is,

a) 2 b) 3 c) 0 d) i

Q.2 A) Define the following (Any five out of seven questions) (05)

(1) Ordinary differential equation.
(2) Cauchy-Euler differential equations.
(3) Power Series
(4) Harmonic Functions
(5) Fourier Series
(6) Singular Points
(7) Continuity of complex function .

B) Answer the following (Any five out of seven questions) (05)

(1) Write the solution of partial differential equation $z = px + qy + \sqrt{sinp - cosq}$
(2) Write $J_0(x)$
(3) Express x^2 in terms of Legendre polynomials.
(4) What is $Im(z)$ of z^2?
(5) Give any example of a second order ordinary differential equation.
(6) Write half range sine series.
(7) Write value of $J_{1/2}(x)$

Q.3 Do as directed. (Any five) (10)

(1) Express $f(x) = x$ as Fourier sine series in the interval (0,π)
(2) Write ordinary and singular points for the differential equation
$$(x^2+4)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$$
(3) Solve the differential equation $y'' + 5y' + 6y = 0$
(4) Check whether $u = x^2 - y^2 + 2xy$ is harmonic function or not?
(5) Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} cosx$
(6) Check whether $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$ is exact or not ?

Q.4 Answer the following. (Attempt any three) (15)

(1) Obtain power series solution for $y' + 2xy = 0$ near an ordinary point.
(2) Examine the continuity of $f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$
(3) Form partial differential equation for the expression ,
$f(x+y+z, x^2+y^2+z^2) = 0$, where f is an arbitrary function.
(4) Solve $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$