Seat No:_____________ **Enrollment No:_____________**

PARUL UNIVERSITY
FACULTY OF AGRICULTURE
B.Tech. Winter 2019 - 20 Examination

Semester: 2 **Date: 12/12/2019**
Subject Code: 20103153 **Time: 10:30 am to 12:30 pm**
Subject Name: Engineering Mathematics - II **Total Marks: 50**

Instructions
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1

A) Fill in the blanks (Each of 0.5 Mark) **(05)**

i) The general integral of $2p + 3q = z$ is _______.

ii) The Wronskian of two functions x and x^2 is _________.

iii) The one dimensional heat equation is defined as _______.

iv) If $u = x^2 + y^2$ is harmonic, then the corresponding analytic function $f(z) =$ _______.

v) The equation $|z| = 2$ represents _______.

vi) If $z = x + iy$ then $|z|$ is _______.

vii) The real part of $f(z) = 3z + 2$ is ________.

viii) The special function $P_n(x)$ is roots of _______ differential equation.

ix) Linear partial differential equation can be solved by _________ method.

x) The Bernoulli equation $\frac{dy}{dx} + p(x)y = Q(x)y^n$ can be reduced to separation of variable equation by using the substitution $n =$ _________ .

B) Multiple Choice Questions (Each of 0.5 Mark) **(10)**

i) $\left[1 + (y')^2\right]^{\frac{1}{2}} = y''$ is a differential equation with order _______ and degree ____.

a)	2, 2	b)	3, 2
c)	2, 4	d)	4, 2

ii) Which of the following equation is not a differential equation?

a)	$(y^2 - x^2)dx + 2xydy = 0$	b)	$(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$
c)	$x + 2y = 0$	d)	$ye^x dx + (2y + e^x)dy = 0$

iii) A complete integral of the second order linear differential equation is having ________ arbitrary constants.

a)	3	b)	2
c)	1	d)	0

iv) Which of the following is a suitable y_p for the differential equation

$y'' + 2y = x^2$ in undetermined coefficient method?

a)	$y_p = Ax + B$	b)	$y_p = Ae^{-2x}$
c)	$y_p = Ax^2 + Bx + c$	d)	None of these

v) Which of the following is not a solution of Bessel's equation?

a)	$J_n(x)$	b)	$P_n(x)$
c)	$J_{-n}(x)$	d)	None of these

vi) The differential equation $x\frac{dy}{dx} + 1 = x$, $x(0) = 1$ has __________.

a)	a unique solution	b)	two solutions
c)	infinite number of solutions	d)	no solution

vii) Let W is Wronskian. If ________, then the set is linearly dependent.

a)	$W = 0$	b)	$W^2 \neq 0$
c)	$W \neq 0$	d)	$none\ of\ these$

viii) The value of $\sin n\pi$ when $n = 200$ is ____________.

a)	200	b)	2
c)	1	d)	0

ix) The period of trigonometric function $\sin x$ is _________.

a)	2π	b)	0
c)	π	d)	3π

x) __________ is an even function of $'t'$.

a)	t^3	b)	$\sin t$
c)	$\cos t$	d)	t^5

xi) Which of the following is Dirichlet's condition?

a)	$f(x)$ is not periodic.	b)	$f(x)$ is infinite.
c)	$f(x)$ is single valued function.	d)	None of these

xii) The value of $\int_0^{2\pi} \cos 3x \sin x \ dx =$____________.

a)	0	b)	π
c)	$-\pi$	d)	None of these

xiii) The value of $\left|e^{i\pi}\right|$ is ________.

a)	1	b)	$1+i$
c)	-1	d)	None of these

xiv) If $f(z) = z + \bar{z}$, then imaginary part of $f(z)$ is ________.

a)	$2x$	b)	$2y$
c)	0	d)	None of these

xv) The solution of $xp + yq = z$ is_________.

a)	$f\left(x^2, y^2\right) = 0$	b)	$f(xy, yz) = 0$
c)	$f(x, y) = 0$	d)	$f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

xvi) If $u = x^2 + 4t^2$ is a solution of $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$, then $c =$_________.

a)	1	b)	2
c)	0	d)	None of these

xvii) The partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6$ is_________.

a)	Elliptic	b)	hyperbolic
c)	Parabolic	d)	None of these

xviii) Which of the following is one dimensional Laplace equation?

a)	$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$	b)	$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
c)	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$	d)	None of these

xix) Which of the following partial differential equation has the general solution $pq = 1$?

a)	$z = x + y + c$	b)	$z = \frac{1}{b}x + ay + c$
c)	$az + b = a^2x + y$	d)	None of these

xx) The degree and order of the partial differential equation

$p^2 + q^2 = z$ are ________.

a)	$1, 2$	b)	$2, 1$
c)	$1, 1$	d)	None of these

Q.2A) Define the following (Any five out of seven questions) **(05)**

(1) Which is necessary condition for an exact differential equation?

(2) State formula of solution of Bessel's differential equation.

(3) Find singular points of the equation $(1 + x^2)y'' - 3xy' - 2y = 0$.

(4) Find product of $2 + 3i$ and $-1 + i$.

(5) State Cauchy-Riemann's equation.

(6) Give example of non-linear partial differential equation.

(7) Write example of second order first degree partial differential equation.

Q.2 B) Answer the following (Any five out of seven questions) **(05)**

(1) Define linear differential equation.

(2) Define particular solution of a differential equation.

(3) Define Cauchy-Legendre's differential equation with variable coefficient.

(4) Define complex number.

(5) Define half range Fourier series.

(6) Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$.

(7) State one dimensional Heat equation.

Q.3 Write Short notes (Any five out of six questions) **(10)**

(1) Solve $y^2 y' = 2x^2$.

(2) Solve $(D^2 + 2D + 1)y = 0$.

(3) Show that $u = 2x - x^3 + 3xy^2$ is harmonic.

(4) Find the Fourier sine series of $f(x) = x$ in $0 < x < \pi$.

(5) Solve $p^2 + q^2 = x + y$.

(6) Form a partial differential equation for the equation $z = (x^2 + a)(y^2 + b)$.

Q.4 Long Questions (Any three out of four questions) **(15)**

(1) Solve $y'' + 2y' + y = 2\cos 2x + 3x + 2 + 3e^x$.

(2) (i) Express half range cosine series of $f(x) = e^x$ in the interval $(0, \pi)$.
(ii) Check whether the function $f(z) = |z|^2$ is analytic or not?

(3) Obtain the Fourier series of the function, $f(x) = \frac{\pi - x}{2}, 0 \le x \le 2\pi$ and prove that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

(4) Solve $px^2(y - z) + qy^2(z - x) = (x - y)z^2$.