Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF AGRICULTURE

B.Tech. Agriculture Winter 2019 - 20 Examination

Semester: 1 **Date: 28/11/2019**
Subject Code: 20103108 **Time: 10.30 am to 1.00 pm**
Subject Name: Engineering Mathematics-1 **Total Marks: 50**

Instructions

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Do as Directed

A) Fill in the blanks (Each of 0.5 Mark) **(05)**

i) If $J = \frac{\partial(u,v)}{\partial(x,y)} = 3 \; then \;\; J' = \frac{\partial(x,y)}{\partial(u,v)}$ = ___________

ii) Convert Quadratic form into matrix form $x^2 + 2xy + y^2$ *is* _____________

iii) The value of $\lim_{x\to 0} x sin\frac{1}{x}$ *is*________

iv) For Z = 1+ i then argZ =_____________

v) State the 1st order Euler's theorem for Homogenous function.

vi) The 5th term of the series $1 + \frac{2}{3} + \frac{4}{9} + \cdots$ ... is__________

vii) An interval (-R,R) in which a power series converges is called___________

viii) Find the rank for A = $\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$

ix) Find $f_x(1,2)$ for $f(x,y) = x^2 - 3xy + 4y^2$.

x) The series $\sum \frac{1}{n^p}$ is convergent for which value of p ?

B) Multiple Choice Questions (Each of 0.5 Mark) **(10)**

i) If x = u+v and y = u-v the $\frac{\partial(x,y)}{\partial(u,v)}$ =______

(a) -2 (b) u (c) v (d) uv

ii) Which of the following are homogeneous functions in x and y ?

(a) $f(x,y) = x^2 + y^2 + \tan(xy)$ (b) $f(x,y) = \tan^{-1}\left(\frac{x^2 - y^2 + 2xy}{2xy}\right)$

(c) $f(x,y) = x^{-3} - y^{-3} + \frac{1}{xy}$ (d) $u = g\left(\frac{1}{xy} + 1\right)$

iii) If eigen values of a 3×3 matrix are 1,8 then the eigen values of $4A^{-1}$ will be

(a) 4, $\frac{1}{2}$ (b) 1,2 (c) 4,2 (d) 4 , 1

iv) The value of x for which the graphs of y = x and $y^2 = 4x$ intersect are

(a) 0 and 4 (b) 4 and -4 (c) 0 and -4 (d) 4 and 4.

v) If a 3×3 matrix A is diagonalizable then which one of the following is true?

(a) A has 2 distinct eigenvalues. (b)A has 2 linearly independent eigenvalues.

(c) A has 3 linearly independent eigenvalues. (d) none of these.

vi) The series $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \cdots + \frac{x^n}{(2n-1)2n}$ is

(a) power series (b)p series (c)alternative series (d)geometric series

vii) The characteristic equation of the matrix $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ is

a) $\lambda^2 - 3\lambda + 2 = 0$ (b) $\lambda^2 + 3\lambda + 2 = 0$

(c) $\lambda^2 - 3\lambda - 2 = 0$ (d) $\lambda^2 - 3\lambda - 1 = 0$

viii) The improper integral $\int_0^\infty \frac{1}{x} dx$ is which type of integral?

(a) Type-I (b) Type-II (c) neither type-I nor Type-II (d) none

ix) $\int \sec^2 x dx =$ _____

(a) tanx (b) -tanx (c) cotx (d) -cotx

x) The coefficient of x^5 in the expansion of e^x is

(a) $\frac{1}{5!}$ (b) $-\frac{1}{5!}$ (c) $\frac{1}{4!}$ (d) $\frac{1}{5}$

xi) The value of i^{10} is

(a) i (b) $-i$ (c) 1 (d) -1

xii) $if\ f(x,y) = x^2 + 2y^2\ then\ \frac{\partial^2 f}{\partial x \partial y} =$ ___

(a) 2 (b) 0 (c) 1 (d) 4

xiii) Which of the following matrix is in row echelon form

(a) $\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

xiv) The value of $\lim_{x\to\infty} \left(1 + \frac{1}{n}\right)^n$ is

(a) $\frac{1}{e}$ (b) 0 (c)∞ (d)e

xv) The eigen values of $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ are

(a) 1,1,1 (b) 1,2,2 (c) 2,3,3 (d) 1,2,3

xvi) If $= r cos\theta$, $y = r sin\theta$ then $\frac{\partial r}{\partial x} =$

a) $\frac{x}{r}$ (b) $cos\theta$ (c) r (d) Does not exist

xvii) The integral $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

(a) Odd function (b) even function

(c) neither odd nor even function (d) none of these.

xviii) The value of $\lim_{x\to 2} \frac{x^2-x-2}{x^2-4}$ is

(a) $\frac{3}{4}$ (b) 0 (c) ∞ (d) 1

xix) A point (a, b) is said to be saddle point if at (a, b)

$(a) rt - s^2 > 0$ (b) $rt - s^2 = 0$ (c) $rt - s^2 < 0$ (d) $rt - s^2 \neq 0$

xx) The value of $JJ^* \ is$

a) 1 (b) 0 (c) ∞ (d)-1

Q.2 Do as Directed

A) Define the following (Any five out of seven questions) (05)

(1) Define D'Alembert'S Ratio Test.

(2) Define Modified Euler's theorem for a function of two variable.

(3) Define Index in canonical form.

(4) Define Trace of matrix.

(5) State De-moivre's theorem.

(6) Define Beta function.

(7) State Roll's theorem.

B) Answer the following (Any five out of seven questions) (05)

(1) Find the n^{th} term of the series $\frac{2}{1} + \frac{3}{8} + \frac{4}{27} + \frac{5}{64}$ ...

(2) Find the value of $\lim_{x\to\infty} \frac{1+2+3+\cdots x}{2x^2}$.

(3) Find the equation of normal line to the surface xyz=6 at (1,2,3).

(4) Write formula chain Rule for u = f(x,y) where x = f(t) and y = f(t).

(5) Find the real part of 2+3i

(6) Write the quadratic form for $[x \quad y]\begin{bmatrix}3 & 2\\ 2 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$

(7) If $A = \begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$ then find A^{-1}

Q.3 Write Short notes (Any five out of six questions) (10)

(1) Find the Stationary values of the function $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

(2) $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

(3) The graph of y = x^2 between x = 1 and x = 2 is rotated around the X-axis. Find the volume of a solid.

(4) If $x^3 + y^3 = 3axy, find \frac{dy}{dx}$.

(5) Find the sum of infinite series $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \ldots ..$

(6) Evaluate $\int_1^\infty \frac{dx}{x^2}$

Q.4 Long Questions (Any three out of four questions) (15)

(1) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

(2) Solve the following system by gauss- Elimination method

x + y + 2z = 9,

2x + 4y -3z =1,

3x +6y - 5z = 0

(3) Prove that $$\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5} = 1$$

(4) Test the converges $\frac{2n^2 + 3n}{\sqrt{5 + n^5}}$