Seat No: _____________ Enrollment No: ___________________

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

M.Tech., Winter 2017 - 18 Examination

Semester: 1 **Date: 28/12/2017**

Subject Code: 03204102 **Time: 02:00PM to 04:30PM**

Subject Name: Statistical Signal Analysis **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 A) What is significance of Cumulative Distribution Function (CDF)? Show that Cumulative Distribution function (CDF) of random variable is non-decreasing function. **(05)**

B) A lot of 100 semiconductor chips contain 20 that are defective. Two chips are selected at random, Without replacement, from the lot. **(05)**

(i) What is the probability that the first one selected is defective?

(ii) What is the probability that the second one selected is defective given that the first one was defective?

(iii) What is the probability that both are defective?

C) Suppose the joint pmf of a bivariate random variable (X,Y) is given by **(05)**

$$P_{XY}(x_i,y_j) = \begin{cases} \frac{1}{3} & (0,1),(1,0),(2,1) \\ 0 & , \quad otherwise \end{cases}$$

(a) Are X and Y independent?

(b) Are X and Y uncorrelated?

Q.2 **Answer the following questions**. (Attempt any three) (Each five mark) **(15)**

A) The pdf of a continuous random variable X is given by

$$f_x(x) = \begin{cases} \frac{1}{3} & , 0 < x < 1 \\ \frac{2}{3} & , 1 < x < 2 \\ 0 & , otherwise \end{cases}$$

Find the corresponding cdf $F_x(x)$ and sketch $f_x(x)$ and $F_x(x)$

B) What is characteristics function? State its uses.

C) Sketch the ensemble of the random process

$$x(t) = a\cos(wt + \theta)$$

where **w** and θ are constants and **a** is an RV uniformly distributed in the range (-A,A).

(a) Just by observing the ensemble, determine whether this is a stationary or a non stationary process.

(b) Determine $\overline{x(t)}$ and $R_x(t_1,t_2)$ for this random process and determine whether this is a wide-sense stationary process.

D) A Modem transmits a binary iid equiprobable data sequence as follws: transmit a binary 1, the Modem transmits a rectangular pulse of duration T seconds and amplitude 1: to transmit a binary 0, it transmits a rectangular pulse of duration T seconds and amplitude -1. Let X(t) be the random process that results. Is X(t) wide sense cyclostationary?

Q.3 A) The joint pdf of random variable (X,Y) is given by **(07)**

$$f_{XY}(x,y) = \begin{cases} kxy\ , & 0 < x < 1, 0 < y < 1 \\ 0, & otherwise \end{cases}$$

Where k is a constant.

(a) Find the value of k.

(b) Are X and Y independent?

(c) Find the conditional pmf's $P_{\frac{Y}{X}}\left(\frac{y_j}{x_i}\right)$ and $P_{\frac{X}{Y}}\left(\frac{x_i}{y_j}\right)$ for the random variable (X,Y).

B) Show that a simple random walk process X (n)={Xn , n ≥ 0} is a Markov chain. Find its one-steptransition probabilities. **(08)**

OR

B) Let normal random variable , X=N(μ ;σ^2) , If its pdf is given by **(08)**

$$f_X(x) = \frac{1}{\sqrt{2\pi}\ \sigma}\ e^{-(x-\mu)^2 / (2\sigma^2)}$$

Verify mean and variance of the normal random variable X are

$$\mu_X = E(X) = \mu$$
$$\sigma_X{}^2 = \mathrm{Var}(X) = \sigma^2$$

Q.4 A) If a random process X(t) is represented by a Karhunen-Loeve expansion **(07)**

$$X(t) = \sum_{n=1}^{\infty} X_n \phi_n(t) \qquad 0 < t < T$$

And Xn's are orthogonal, show that $\emptyset_n(t)$ must satisfy integral equation

$$\int_0^T R_X(t, s)\phi_n(s)\, ds = \lambda_n \phi_n(t) \qquad 0 < t < T$$

OR

A) Find the Karhunen-Loeve expansion of the Wiener process X(t). **(07)**

B) Explain Markov Process in detail. **(08)**