Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Summer 2017-18 Examination

Semester: 2 **Date: 11/05/2018**

Subject Code: 11206153 **Time: 10:30AM TO 01:00PM**

Subject Name: Advanced Abstract Algebra **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions **(08)**

(a) If f is onto homomorphism from G to G' then prove that $\frac{G}{Ker\ f} \cong G'$

(b) Prove that in usual notation, $o(G) = o\big(Z(G)\big) + \sum_{a \notin Z(G)} \frac{O(G)}{o(N(a))}$

Q.1. B) Answer the following questions (Any two)

(a) Answer the following questions **(04)**

1. Prove that $Z(G)$ is a normal subgroup of group G.
2. Check whether $Z_2 \times Z_3$ is cyclic or not?

(b) Show that the number of generators of an infinite cyclic group is two **(04)**

(c) Prove that the order of each subgroup of any finite group divides the order of a group. **(04)**

Q.2. A) Answer the following questions.

(a) Prove that a group homomorphism $\varphi: G \to G'$ is a one one map if and only if $Ker(\varphi) = \{e\}$ **(04)**

(b) Show that G = {0, 1, 2, 3, 4} is an abelian group under the operation $+_5$. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) Short note/ Multiple choice questions. (Each of 01 marks) **(03)**

1. HK is a sub-group of G if and only if

   (a) $HK = KH$ (b) $HK \subset KH$ (c) HK ⊃ KH (d) $HK \neq KH$

2. Let G be a group of order 15 then the number of 3 sylow subgroups of G is

   (a) 0 (b) 1 (c) 3 (d) 5

3. A field is

   (a) vector space (b) integral domain (c) division ring (d) commutative division ring

(b) Let $R^2 = R \times R = \{(a, b); a \in R, b \in R\}. T : R^2 \to R^2$ such that $T_{(a,b)}(x, y) = (x + a, y + b)$. Let G = $\{T_{(a,b)}\ ; a, b\ \epsilon R\}$ is an abelian group. **(03)**

(c) Prove that every group of prime order is cyclic. **(03)**

Q.3. A) Answer the following questions **(08)**

(a)Prove that finite integral domain is a field

(b) Show that set M of all matrices of the type $\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$ with a and b as integers forms a left ideal of the ring R of all 2×2 matrices with elements as integers. But it does not form a right ideal.

Q.3. B) Answer the following questions (Any two)

(a) Answer the following questions (Each of 02 marks) **(04)**

1. Let a and b be arbitrary elements of a ring R whose characteristic is 2 and $ab = ba$. Then show that $(a + b)^2 = a^2 + b^2$
2. If U is an ideal of the ring R and 1∈ U , Prove that $U = R$.

(b) State and Prove Sylow's first theorem. **(04)**

(c) Prove that the necessary and sufficient conditions for a non empty subset S to be subring of a ring $< R, +, . >$ are (i) $a, b\ \epsilon\ S \Rightarrow a - b\ \in S$ and (ii) $a, b\ \in S \Rightarrow ab\ \epsilon\ S$ **(04)**

Q.4. A) Answer the following questions.

(a) Answer the following questions (Each of 02 marks) **(04)**

1. Prove that if $a, b\ \epsilon\ R$ then prove that $(a + b)^2 = a^2 + ba + ab + b^2$, where by x^2we mean

xx and R is a ring.

2. Prove that the only idempotent elements of an integral domain are 0 and 1.

(b) Let $* \, and \, \Delta$ be two binary compositions defined in the set I of all integers given as $a * b = a + b - 1$ and $a\Delta b = a + b - ab, \forall a, b \epsilon I$. Show that $< I, *, \Delta >$ is a commutative ring with unity **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Define following terms (Each of 01 marks) **(03)**

1. Factors of a Subnormal Series
2. Length of Subnormal Series
3. Refinement of Subnormal Series

(b) Show that the set Q of rational number is not an ideal of the ring of real numbers $< R, +, . >$ **(03)**

(c) Prove that intersection of two ideals is again an ideal **(03)**