Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Summer 2017-18 Examination

Semester: 2 **Date: 09/05/2018**
Subject Code: 11206152 **Time: 10:30 am to 1:00 pm**
Subject Name: Applied Partial Differential Equations **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Attempt any one **(08)**

(a) Using the method of separation of variables, solve the following equation:

$$\frac{\partial^2 y}{\partial t^2} - \frac{c^2 \partial^2 u}{\partial x^2} = 0; 0 < x < l, t > 0,$$
$$y(x,0) = f(x); 0 \le x \le l,$$
$$\frac{\partial y}{\partial t}(x,0) = g(x); 0 \le x \le l, y(0,t) = y(l,t) = 0; t > 0$$

(b) Solve the BVP $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, 0 < x < L, u(0,t) = 0, u(L,t) = 0, t > 0$

$$u(x,0) = \begin{cases} 1, 0 < x < \frac{L}{2} \\ 0, \frac{L}{2} < x < L \end{cases}$$

Q.1. B) Answer the following questions. (Any two) **(08)**

(a) Reduce the equation $\frac{\partial^2 u}{\partial x^2} - x^2\frac{\partial^2 u}{\partial y^2} = 0$ to a canonical form.

(b) State and prove Superposition principle.

(c) If the function R, P and Z contains y but not x. Show that the solution of $Rr + Pp + Zz = w$ can be obtained from a second order ODE with constant coefficients .Hence solve $yr + (y^2 + 1)p + yz = e^x$

Q.2. A) Answer the following questions.

(a) 1. Classify the PDE $yp - xq = xyz + x$ in Linear, Semilinear and Quasilinear. **(04)**
2. Eliminating arbitrary function from the partial differential equation for $z = xy + f(x^2 + y^2)$

(b) Solve the equation $(D^2 - 2DD')z = e^{2x} + x^2y$ **(04)**

Q.2. B) Answer the following questions (Any two) **(06)**

(a) Verify the PDE $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$ is satisfy by $z = \frac{1}{x}Q(y-x) + Q'(y-x)$,where Q is an arbitrary function

(b) Prove that $F(D,D')\, e^{ax+by} = F(a,b)e^{ax+by}$, where a & b are constants.

(c) Find the particular integral of $(D^2 + 2DD' + D'^2)z = \sin(2x + 3y)$

Q.3. A) Find the solution of Dirichlet problem for the upper half plane which is defined as $u_{xx} + u_{yy} = 0; -\infty < x < \infty; y > 0$, $u(x,0) = f(x);\ -\infty < x < \infty$ with the condition that u is bounded as $y \to \infty$, u *and* u_x vanish as $|x| \to \infty$ **(08)**

Q.3. B) Answer the following questions (Any two)

(a) Solve the differential equation $z^2(p^2z^2 + q^2) = 1$ **(04)**

(b) Find the complete integral of $p^2 + q^2 = x + y$ **(04)**

(c) If $\beta_r D' + \gamma_r$ is a factor of $F(D,D')$ & $\phi_r(\xi)$ is an arbitrary function of the single variable ξ if $\beta_r \neq 0$ then $u_r = \exp(\frac{-\gamma_r y}{\beta_r})\phi_r(\beta_r x)$ is a solution of $F(D,D')z = 0$. **(04)**

Q.4. A)

(a) **Fill in the blanks with correct answer.** **(04)**

1 The complementary function for the equation $\left(D^2D' + 2DD'^2 + D'^3\right)z = \frac{1}{x^2}$ is ________.

2 The solution of $\frac{\partial^2 u}{\partial y^2} = 0$is ____________.

(b) Solve the equation $\frac{y-z}{yz}\ p = \frac{z-x}{zx}\ q = \frac{x-y}{xy}$ by Lagrange's Method. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Choose the correct option for the following multiple choice questions. **(03)**

1 The longitudal vibrations of a bar satisfies the equation

(a) $u_{xx} = u_{tt}$ (b) $u_{xx} + u_{yy} = 0$

(c) $u_{xx} = u_t$ (d) $u_{xx} = u_{yy}$

2 The solution $z = ax + by + ab$ of the PDE $z = px + qy + pq$ is a

(a)General Integral (b)Singular Integral

(c) Complete Integral (d)Partial Integral

3 The PDE $xp - xyp = xz^2$is a

(a)Linear PDE (b)Semi-linear PDE

(c)Quasi-linear PDE (d)non Linear PDE

(b) Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ **(03)**

(c) Find the characteristic equation of the PDE $u_x + 2u_y = u\ , u(0,y) = y\ on\ \Gamma = \{(0,r)\}$. **(03)**