Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Summer 2017-18 Examination

Semester: 2 **Date: 07/05/2018**
Subject Code: 11206151 **Time: 10:30 am to 1:00 pm**
Subject Name: Complex Analysis **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions **(08)**

(a) Prove that C.R. equations at a point $z_0 = x_0 + iy_0$ are necessary conditions for $f(z) = u(x,y) + v(x,y)$ be differentiable at Z_0.

(b) Check whether the following function $f(z)$ continuous or not at point $z = 0$.

$$f(z) = \begin{cases} \dfrac{\bar{z}^3}{Re(z^2)} & z \neq 0 \\ 1 & z = 0 \end{cases}$$

Q.1. B) Answer the following questions (Any two)

(a) Answer the following questions. (Each of 2 Marks) **(04)**

1. Find the Principal Value of $(-i)^i$
2. Check the analytic behavior of the function $f(z) = \bar{z}$

(b) Explain the differentiability of $f(z) = \frac{z-\bar{z}}{2}$ at $z = i$ using the definition. **(04)**

(c) Determine an analytic function $f(z)$, if $Re[f(z)] = \arg(z)$. **(04)**

Q.2. A) Answer the following questions.

(a) Answer the following questions. (Each of 2 Marks) **(04)**

1. If the contour c is along the upper half of the circle $|z| = 3$ from $z = -3$ to $z = 3$ then without finding the actual value of integral prove that $\left|\int_c \frac{\sqrt{z}}{z^2+1} dz\right| \leq \frac{3\sqrt{3}\pi}{8}$
2. If s is a square of side "a" and z_0 be any point lies completely inside s then $\left|\int_s (z - z_0)\right| \leq 4\sqrt{2}\ A$, where A is area of s.

(b) Suppose that (i) c is a simple closed contour, describe in the counterclockwise direction. **(04)**
(ii) $c_k (k = 1,2,3 \dots .n)$ are simply closed contours interior to c, all describe in clockwise direction that are disjoint and whose interior have no points in common. If a function f is analytic on all of these contours and throughout the multiple connected domain consisting of the points inside c and exterior to each c_k then prove that $\int_c f(z)dz + \sum_{k=1}^{n} \int_{c_k} f(z)dz = 0$

Q.2. B) Answer the following questions (Any two)

(a) Choose the correct option for the following multiple choice questions. (Each of 01 marks) **(03)**

1. The value of the integral $\int_C \frac{dz}{Z+2}, C: |z| = 1$ is
   (a) $2\pi i$ (b) $-2\pi i$ (c) $4\pi i$ (d) 0
2. If $w = f(z) = u(x,y) + iv(x,y)$ is analytic then $f'(z)$ equal to
   (a) $u_x - iu_y$ (b) $u_x - iv_x$ (c) $v_y - iv_x$ (d) none of these
3. The Cauchy integral theorem states "If $f(z)$ is analytic in simply connected domain D, then $\oint_c f(z)dz = 0$ on every simple close path C in D". The condition of analytic in this theorem is
   (a) necessary (b) necessary and sufficient (c) sufficient (d) none of these

(b) Evaluate $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)} dz; c: |z| = 2$ **(03)**

(c) Evaluate $\oint_C \frac{e^z}{z^2+1} dz\ ; c: |z| = 2$ **(03)**

Q.3. A) Answer the following questions **(08)**

(a) State and Prove Cauchy's Residue theorem

(b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region (i) $|z| < 1$, (ii) $|z| > 2$

Q.3. B) Answer the following questions (Any two)

(a) Answer the following questions (Each of 02 marks) **(04)**

1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty}(logn^n)z^n$
2. Find the residue of the function $f(z) = \frac{\tan h\, z}{z^2}$

(b) Using the Residue theorem evaluate $\int_0^{2\pi}\frac{d\theta}{1-2acos\theta+a^2}$, where $a^2 < 1$ **(04)**

(c) Determine the map of rectangle region $a \leq x \leq b, c \leq y \leq d$; $a, b, c, d > 0$ in the first quadrant of Z-plane under the transformation $w = \frac{1}{z}$ **(04)**

Q.4. A) Answer the following questions.

(a) Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center zero. **(04)**

(b) Prove that under the bilinear transformation cross ratio of four points remain invariant **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Short note/ Multiple choice questions. (Each of 01 marks) **(03)**

1. for the function $f(z) = \frac{1-e^{-z}}{z}$, the point $z = 0$ is
   (a) an essential singularity (b) a pole of order 0 (c) a pole of order one (d) a removable singularity
2. Which of the following function does represent the series $\sum_{n=0}^{\infty}\frac{z^n}{n!}$ for $|z| < \infty$?
   (a) sinz (b) cosz (c) e^z (d) log(1+ z)
3. The fixed point of the mapping $w = \frac{5z+4}{z+5}$ are
   (a) 2, 2 (b) 2, -2 (c) -2, -2 (d) -4/5, 5

(b) Determine bilinear transformation which maps z = 1, 0, -1 in to the points w = 0, -i, ∞ respectively. **(03)**

(c) Explain the conformal behavior of $w = \bar{z}$ at $z = 0$. **(03)**