Seat No: _____________ **Enrollment No: _____________**

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc.Summer 2018 - 19Examination

Semester: 4 **Date: 03/04/2019**

Subject Code: 11206252 **Time: 2:00 pm to 4:30 pm**

Subject Name: Finite Element Method **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions. **(08)**

(a) Construct the weak form for a nonlinear equation $-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f = 0 \; for \; 0 < x < 1$

$$\left(u\frac{du}{dx}\right)_{x=0} = 0, u(1) = \sqrt{2}$$

(b) Write an Imposition of Boundary conditions.

Q.1. B) Answer the following questions (Any two)

(a) Draw a Flow chart of the computer program FEM2D **(04)**

(c) Let $u \; and \; uh$, respectively, be the solution of$a(u, v) = L(v) \forall v \in V, and \;\; a(u_h, v_h) = L(v_h) \forall v_h \in V_h$. Let all the conditions for the Lax - Milgram Theorem hold. Then there is a constant C independent of the discretization parameter h such that $\| u - u_h \| V \leq C \inf_{x \in V_h} \| u - X \| V$ **(04)**

(c) Let V be an element in $H^2(I)$ which vanishes at $x = 0$. Let $I_h v$ be its nodal interplant which is defined as$I_h v(x) = \sum_{i=1}^{N} v(x_i)\phi(x_i)$. Then the following estimates $\| (v - I_h v)' \| \leq C_1 h \| v'' \|$, and $\| v - I_h v \| \leq C_2 h^2 \| v'' \|$ **(04)**

Q.2. A) Consider the differential equation $-\frac{d^2u}{dx^2} = cos\pi x \; for \; 0 < x < 1$ subject to the boundary conditions$u(0) = 0, u(1) = 0$. Determine a three parameter solution, with trigonometric functions using the Ritz method. **(08)**

Q.2. B) Answer the following questions (Any one)

(a) Give a one parameter Galerkin solution of the equation $-\nabla^2 u = 1 \; in \; \Omega$ **(06)**
$u = 0 \; on \; \Gamma$ use trigonometric functions.

(b) Consider the differential equation $-\frac{d^2u}{dx^2} - u + x^2 = 0 \;\; for \; 0 < x < 1$ with two sets of boundary conditions: set 1: $u(0) = 0, u(1) = 0$ **(06)**

set 2: $u(0) = 1, \left(\frac{du}{dx}\right)_{x=1} = 1$. Determine a three parameter solution, with trigonometric functions using the Petrov Galerkin Method.

Q.3 A) Answer the following questions. **(08)**

(a) Explain PetrovGalerkin Method.

(b) Explain Galerkin method.

Q.3. B) Answer the following questions (Any one)

(a) Find a one parameter approximation solution of nonlinear equation **(08)**

$-2u\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2 = 4 \; for \; 0 < x < 1$ subject to the boundary conditions $u(0) = 0 \; and \; u(1) = 1$ with using least squares methodwith weight function $w = 1$.

(b) Compute the coefficient matrix and the right hand side of the N parameter Ritz approximation of the equation $-\frac{d}{dx}\left[(1 + x)\frac{du}{dx}\right] = 0 \; for \; 0 < x < 1$ **(08)**
$u(0) = 0, u(1) = 1$ use algebraic polynomials for the approximation functions. Specialize your result for $N = 2$.

Q.4 A) Consider the differential equation $-\frac{d^2u}{dx^2} - u + x^2 = 0 \;\; for \; 0 < x < 1$ for the boundary conditions $u(0) = 0, \left(\frac{du}{dx}\right)_{x=1} = 1$ by using finite element method for the uniform mesh of three linear elements. **(08)**

Q.4. B) Answer the following questions (Any one)

(a) Let u be a solution of $a(u,v) = (f,v)\ \forall v \in V\ with\ u \in H^2(I) \cap H_0^1(I)$. Let u_h be a solution of $a(u_h, v_h) = (f, v_h), v_h \in V_h$. Then there is a constant C independent of h such that the error $e = (u - u_h)$satisfies $\| e' \| \leq C_h \| u'' \|\ and\ \| e \| \leq C_h^2 \| u'' \|$. **(06)**

(b) Consider the differential equation $-\frac{d^2u}{dx^2} = cos\pi x\ for\ 0 < x < 1$ subject to the boundary conditions$u(0) = 0, \left(\frac{du}{dx}\right)_{x=1} = 0$ by using finite element method for the uniform mesh of three linear elements. **(06)**