Seat No:_____________ **Enrollment No:_____________**

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Summer 2018-19 Examination

Semester: 2 **Date: 05/04/2019**
Subject Code: 11206153 **Time: 10:30am to 1:00pm**
Subject Name: Advanced Abstract Algebra **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. Answer any ONE of the following. **(08)**
A)

(a) Let G be a finite group such that p^m divides $o(G)$ and p^{m+1} does not divide $o(G)$, where p is a prime number and m is a positive integer $then$ prove that G has subgroups of order $p, p^2, \dots, p^m$.

(b) (i)Let H and K be finite groups of G ,then prove that

$$o(HK) = \frac{O(H)O(K)}{O(H \cap K)}$$

(ii) If $o(H) > \sqrt{o(G)}$ and $o(K) > \sqrt{o(G)}$ Using above result prove that $H \cap K \neq \{e\}$

Q.1. Answer the following questions (Any two)
B)

(a) Define cyclic group and prove that "every group of prime order is cyclic". **(04)**
(b) If G is a group with order $11^2 \times 13^2$ *then find the number of 11-sylows subgroups and 13-sylows subgroups. Also check G is a simple group or not.* **(04)**
(c) *Prove that any two right cosets of a subgroup are either disjoint or identical.* **(04)**

Q.2. Answer the following questions.
A) (a) Short note/ Brief note (2x2)/ Fill in the blanks. (Each of 02 marks) **(04)**
1.Prove that $(a^{-1})^{-1} = a$
2. Check Z *is a subfield of* Q *or not?*
(b) Prove A subgroup H of a group G is the normal subgroup of G if and only if each left coset for that is a right coset of H in G. **(04)**

Q.2. Answer the following questions (Any two)
B) (a) Short note/ Multiple choice questions. (Each of 01 marks) **(03)**
1. Check Z_6 *is a integral domain or not. ?*
2.Write normal series of S_4.
3.Normalizer N(a) of a in G is a normal subgroup.(T/F)
(b) Prove in the ring Z of integers the ideal generated by Prime integer is a maximal ideal. **(03)**
(c) Find the isomorphic refinement of the subnormal series $Z \supset 4Z \supset 8Z \supset \{0\}$ *and* $\supset 9Z \supset \{0\}$. **(03)**

Q.3. Essay type/ Brief note (4x2) (Each of 04 marks) **(08)**
A)

(a)State and prove langrange's theorem
(a) If R is a Boolean ring then prove
1. $a + a = 0 \ \forall a \in R$
2. $a + b = 0 \Rightarrow a = b$
3. *Every Boolean ring is commutative.*

Q.3. Answer the following questions (Any two)
B) (a) Short note/ Brief note (2x2)/ Schematically label the figures (2x2) (Each of 02 marks) **(04)**
1.Using homomorphism check the ring $2Z$ *is isomorphic to the ring* $3Z$ *or not?*
2.If $f=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ *and* $g=\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ *solve the equation fh=g.*
(b) Prove **(04)**
$If\ f \in F[x]\ and\ \deg f = 2\ or\ 3\ then\ f(x)\ is\ reducible\ over\ F\ if\ and\ only\ if\ f(x)\ has\ a\ zero\ in\ F.$

(c) If G is an abelian group ,if a,b $\in$ G,such that o(a)=m ,o(b)=n and (m,n)=1 then prove that o(ab)=mn **(04)**

Q.4. A) Answer the following questions.

(a) Short note/ Brief note (2x2)/ Fill in the blanks. (Each of 02 marks) **(04)**

1.Solve the equation $x^2 + 1 = 0$ in Z_5.

2.Prove that the group ({1,2,3,4},$\times_5$) is cyclic.

(b) The union of two subgroups of a group is a subgroup iff one of them is contained in the other. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Short note/ Multiple choice questions. (Each of 01 marks) **(03)**

1.Show that i is a generator of $(\{i, -i, 1, -1\}, \times)$.

2.Prove that $f(x) = 25x^5 - 9x^4 + 3x^2 - 12$ is irreducible over Q .

3. Find all the units of Z_{12}.

(b) In a group G , for $a, b \in G, O(a) = 5, b \neq e$ and $aba^{-1} = b^2$. Find the order of b. **(03)**

(c) Prove that every finite integral domain is a field. **(03)**