Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Summer 2018-19 Examination

Semester: 2 **Date: 01/04/2019**
Subject Code: 11206151 **Time: 10:30 am to 01:00 pm**
Subject Name: Complex Analysis **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions **(08)**

(a) If $f(z) = \begin{cases} \frac{x^3(1-i)+y^3(1+i)}{x^2+y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ then prove that C-R equations are satisfied at $z = 0$ but function is not differential at $z = 0$.

(b) Check whether the following function $f(z)$ continuous or not at point $= 0$.

$$f(z) = \begin{cases} \frac{Im(z^2)}{|z|} & z \neq 0 \\ 0 & z = 0 \end{cases}$$

Q.1. B) Answer the following questions (Any two)

(a) Answer the following questions. (Each of 2 Marks) **(04)**

1. Find the Principal Value of i^i
2. Check the analytic behavior of the function $f(z) = x^2 + iy^2$

(b) Using Definition of Differentiability prove that $f(z) = |z|^2$ is differentiable only at origin in Z -plane. **(04)**

(c) Prove that, if a function $f(z) = u(x,y) + i\,v(x,y)$ is analytic in domain D, then its component functions u and v are Harmonic in D. **(04)**

Q.2. A) Answer the following questions.

(a) Answer the following questions. (Each of 2 Marks) **(04)**

1. Without finding the actual value of the integral prove that $\left|\int_C \frac{dz}{z^2-1}\right| \leq \frac{\pi}{3}$, where c is the arc of the circle $|z| = 2$ from $z = 2\ to\ z = 2i$ in the first quadrant of Z- plane.
2. If c is any closed contour then prove that $\int_c z\,dz = 0$.

(b) If f is analytic throught a simply connected domain D, then prove that $\int_c f(z)\,dz = 0$ for every closed contour c lying in D. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) Choose the correct option for the following multiple choice questions. (Each of 01 marks) **(03)**

1. The integral $\oint_{|z|=2} \frac{cosz}{z^3} dz$ equals

   a). πi b). $-\pi i$
   c). $2\pi i$ d). $-2\pi i$

2. The function $f(z) = sinz$ is differentiable at

   a). z =0 (b) z ≠ 0 (c) nowhere (d) everywhere

3. For second order zero

   (a) $f(z_0) = f'(z_0) = 0\ and\ f''(z_0) \neq 0$ (b) $f(z_0) = 0, f'(z_0) \neq 0\ and\ f''(z_0) \neq 0$
   (c) $f(z_0) = f'(z_0) = f''(z_0) = 0$ (d) none of these

(b) Evaluate $\oint_C \frac{2z^3+z^2+4}{z^4+4z^2} dz$, where c is the circle $|z-2| = 4$, clockwise **(03)**

(c) Verify Cauchy Gaurset theorem for $f(z) = \frac{1}{z^2}$ along the circle $|z|$=1. **(03)**

Q.3. A) Answer the following questions **(08)**

(a) State and Prove Cauchy's Residue theorem

(b) Using the Residue theorem evaluate $\int_{-\infty}^{\infty} \frac{cosax}{k^2+x^2} dx = \frac{\pi}{k} e^{-ka}$; $a > 0, k > 0$

Q.3. B) Answer the following questions (Any two)

(a) Answer the following questions. (Each of 2 Marks) **(04)**

1. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n+1}{(n+2)(n+3)} z^n$
2. Find the residue of the function $f(z) = cotz$

(b)What is the map of real axis of Z- plane under the transformation $w = \frac{z+i}{z-i}$. Hence deduce the image of upper and lower half of Z- plane under this transformation. **(04)**

(c) Prove that An isolated singular point z_0 of function f is a pole of order m if and only if $f(z)$ can be written in the form $f(z) = \frac{\varphi(z)}{(z-z_0)^m}$, where $\varphi(z)$ is analytic and nonzero at z_0. **(04)**

Moreover Res $f(z)_{z=z_0} = \varphi(z_0)$ if $m = 1$ and Res $f(z)_{z=z_0} = \frac{\varphi^{n-1}(z_0)}{(m-1)!}$ If $m \geq 2$

Q.4. A) Answer the following questions.

(a) Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region (i) $|z| < 2$, (ii) $|z| > 3$ **(04)**

(b) Prove that under the bilinear transformation cross ratio of four points remain invariant **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Choose the correct option for the following multiple choice questions. (Each of 01 marks) **(03)**

1. The invariant point of the transformation $w = \frac{z-1}{z+1}$ are

   (a) z = i (b) z = $\pm i$ (c) z = $\frac{i}{2}$ (d) z = -$\frac{i}{2}$

2. The singularity of the function $\frac{z-sinz}{z^2}$ is

   (a) z = 0 (b) z =2 (c) z = -2 (d) z = 4

3. If $f(z)$ is entire function then the Taylor series is

   (a) convergent for all z (b) divergent for all z (c) constant (d) none of these

(b) Determine the bilinear transform which sends the points z = 1, i, -1 of Z-plane into the points w = 1, -i, -1 respectively. **(03)**

(c) Explain the conformal behavior of $w = z^2$ at $z = 0$. **(03)**