Seat No:______________ **Enrollment No:______________**

PARUL UNIVERSITY
FACULTY OF APPLIED SCIENCE
M.Sc. Winter 2017-18 Examination

Semester: 1 **Date: 28/12/2017**
Subject Code: 11206105 **Time: 02:00pm to 04:30pm**
Subject Name: Number Theory **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Brief note (4x2) (Each of 04 marks) **(08)**

(a) State and prove Fundamental theoram of Divisibility.

(b) If (a,b)=d then `prove that $\exists$ x,y $\in Z$ such that ax+by = d.

Q.1. B) Answer the following questions (Any two)

(a) Short note. (Each of 02 marks) **(04)**

1.Prove that (a-s) / (ab+st) then (a-s) / (at+sb)

2.If a/c and b/c and (a,b) =1 then prove that ab/c

(b) If (a,b) = 1 then prove that (a,bc) = (a,c) **(04)**

(c) prove that common multiple of two non zero integer is also a multiple of their LCM. **(04)**

Q.2. A) Answer the following questions.

(a) **(04)**

1.if the value of T(a) = 2 then find the value of P(a) and S(a).

2.if $a \equiv b \pmod{n}$ *and* $c \equiv d \pmod{n}$ then in usual notation show that

$a + c \equiv b + d \pmod{n}$

(b) if a is square number then show that S(a) is an odd integer. (S(a) is sum of Divisiors) **(04)**

Q.2. B) Answer the following questions (Any two)

(a) Answer the following. **(Each of 01 marks)** **(03)**

1.Define Greatest integer Function.

2. Least common multiple of [a , 0]=_______

a) 0 b) 1 c) a d) does not exists

3. if a(a>2) is a composite number then T(a) is always greater than 2*a.(true/False)

(b) Prove that $[x]+[y] \le [x+y] \le [x]+[y]+1$ (here [x] is greatest integer function.) **(03)**

(c) show that mobious function is multiplicative function. when (a,b)=1 **(03)**

Q.3. A) Brief note (4x2) (Each of 04 marks) **(08)**

(a) State & prove chinese remainder theoram.

(b) prove that $a \equiv b \pmod{n}$ **iff** a and b have same non negative remainders when divided by n.

Q.3. B) Answer the following questions (Any two)

(a) Short note. **(Each of 02 marks)** **(04)**

1.show that congruence is an equivalent relation.

2. if $a_1,a_2,a_3,a_4,\ldots..a_m$ is CRS modulo m and (a, m)=1 then prove that aa_1+m, aa_2+m, aa_3+m, aa_4+m,…..aa_m+m. foarms CRS modulo m, where b is any integer.

(b) State and prove fermat's theoram. **(04)**

(c) solve 12x+15$\equiv$0(mod 45) **(04)**

Q.4. A) Answer the following questions.

(a) Short Questions (Each of 02 marks) **(04)**

1.if $p \neq 2$ *be a prime no. and let a, b be* integers such that p does not divides a and b and p/(a-b) then prove that $\left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$

2.prove that there is no prime p of the form 4k+3 which is express as a sum of two squares.

(b) state and prove unique factorization theoram. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Multiple choice questions. (Each of 01 marks) **(03)**

1.which of the following is not an algebraic number ?

a) 0 b) ½ c) -5 d)none of these

2. which of the following is an prime ideal ?

a) $2\mathbb{Z}$ b) $4\mathbb{Z}$ c)$6\mathbb{Z}$ d) $8\mathbb{Z}$

3. $x \equiv 2 (\bmod 7)$ is a quadratic residue modulo n? (True/False)

(b) Find positive integer solution for 7x+19y=213 **(03)**

(c) prove that positive integer solution of $x^{-1}+y^{-1} = z^{-1}$ ($where$ $(x, y, z) = 1$ has and must have the form **(03)**

x=a(a+b), y=b(a+b) z=ab where a,b > 0 and (a,b)=1