Seat No: _____________ Enrollment No: _____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Winter 2017-18 Examination

Semester: 1 **Date: 26/12/2017**
Subject Code: 11206104 **Time: 02:00 pm to 04:30 pm**
Subject Name: Topology **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Do as directed: (Each of 04 marks) **(08)**

(a) State and prove Lindelof's Theorem.

(b) Show that every closed subspace A of a normal space X is normal.

Q.1. B) Answer the following questions (Any two)

(a) Do as directed: **(04)**

1. Is the product of two path connected spaces necessarily connected?

2. Let X and Y be topological spaces. Describe under which condition a function f : X $\rightarrow$ Y is said to be continuous.

(b) Prove that a closed subset of a compact space is compact. **(04)**

(c) Let A and B be two compact subsets of X. Show A $\cup$ B is compact and provide an example that A $\cap$ B is not compact. **(04)**

Q.2. A) Answer the following questions.

(a) State whether the following statement are true or false with justification **(04)**

1. Every metric space can also be seen as topological space.

2. There are topological space with countably many points, which have uncountable many open sets.

(b) If Y is a subspace of X, a separation of Y is a pair of closed disjoint non-empty sets A& B whose union is Y, neither of which contains a limit point of the other. Prove that the separation is connected if there exists no separation of Y. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) Multiple choice questions. **(03)**

1. Which of the following statements is correct

a) Intersection of two connected sets in a topological space is connected.
b) Countable union of compact sets in a topological space is compact.
c) Closure of a compact sets in a topological space is compact.
d) None of the (a), (b), (c) is correct.

2. An uncountable sets with co-countable topology is

a) Second countable (b) First countable but not second countable
c) Not first countable (d) Separable

3. Let $X = \{a, b, c\}$ & $T = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ then T is

a) The indiscrete topology on X

b) The discrete topology on X.

c) A topology on X which cannot be include by a metric on X.

d) A Hausdorff topology on X.

(b) Provide a counterexample. That is, construct an example of two topological spaces X and Y and a continuous function $f : X \to Y$, where X is connected but Y is not. Make sure to show that, in your example, f is continuous, X is connected, and Y is not connected. **(03)**

(c) Prove that a metric space with Bolzano Wierstrass property is a sequentially compact metric space. **(03)**

Q.3. A) Do as directed: (Each of 04 marks) (08)

(a) Prove that separable metric space is second countable.

(b) Give an example of T_2 space which is not regular.

Q.3. B) Answer the following questions (Any two)

(a) Do as directed: (04)

1. Prove that a topological space X is T_1 if and only if all singletons are closed.

2. Give an example of a separable topological space which is not second countable.

(b) Let X be compact Hausdorff space. Show that X is metrizable if and only if it is second countable. **(04)**

(c) If F and K are subsets of a metric space such that F is closed and K is compact, then $F \cap K$ is compact. **(04)**

Q.4. A) Answer the following questions.

(a) Do as directed: (Each of 02 marks) (04)

1. If I remove finitely many points from the set $S_1 = \{f(x; y) : x^2 + y^2 = 1\}$, is the resulting set connected? Provide an argument supporting your answer.

2. If I remove finitely many points from the set $D = \{f(x; y) : x^2 + y^2 \leq 1\}$, is the resulting set compact? Provide an argument supporting your answer.

(b) Show that the components of X are connected disjoint subspaces of X whose union is X such that each non-empty connected subspace of X intersects only one of them. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Do as directed: (03)

1. Let X be a topological space. Let A be a subset of X. Define the closure A of A.

2. Define when a topological space X is first-countable.

3. Given a subset $A \subset X$, where X is a topological space, carefully define what it means for p to be a limit point of A.

(b) Prove that the union of collection subspaces of X that have a point in common is connected. **(03)**

(c) Prove that composition of two continuous maps on a topological space is continuous. **(03)**