Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Winter 2017-18 Examination

Semester: 1 **Date: 20/12/2017**

Subject Code: 11206102 **Time: 2:00pm to 4:30pm**

Subject Name: Theory of Ordinary Differential Equation **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Consider the problem : $y' = 1 - 2xy, y(0) = 0$ **(08)**

a) Find a solution for this linear equation

b) Consider the above problem on $R: |x| \leq 1/2, |y| \leq 1$. If $f(x,y) = 1 - 2xy$, then show that $|f(x,y)| \leq 2, (x,y) \in R)$. Also show that all successive approximation to the solution exists on $|x| \leq 1/2$ and the graph remains in R

c) Show that f satisfies Lipschitz condition on R with Lipschitz constant $k = 1$ and show that the successive approximations converge for $|x| \leq 1/2$.

d) Show that the approximation $\emptyset_3$ satisfies $|\emptyset \quad (x) - \emptyset_3(x)| \leq 0.01$ for $|x| \leq \frac{1}{2}$.

e) Compute $\emptyset_3$.

Q.1. B) Answer the following questions :(Any two)

(a) Do as directed: **(04)**

1. Examine whether $\emptyset(x) = 1 + e^{-x^2}$ is a solution of the integral equation $y = \int_0^x ty\, dt$ on $(-\infty, \infty)$. or not .

2. Let $f(x,y) = y^2 x$ on $R: |x| \leq 1, y \leq 1$ check whether f satisfies Lipschitz condition on R or not.

(b) Solve the equation $y'' = (y')^2 + 1, \phi(0) = 0, \phi'(0) = 0.$ **(04)**

(c) Find a solution $\vec{\phi}$ of the given system $y_1' = y_1; y_2' = y_1 + y_2$ which satisfies $\vec{\phi}(0) = (1,2)$. **(04)**

Q.2. A) Answer the following questions:

(a) State whether the following statement are true or false with justification. **(04)**

1. The B.V.P $x'' - 4x = e^t, 0 < t < 1,$ with boundary condition $x(0) = x(1) = x'(0) = x'(1) = 0$ is linear.

2. The origin is an unstable critical point for the system $x_1' = 2x_1x_2 + x_1^4; x_2' = 6x_1^2 + x_2^5$.

(b) Let us consider the following continuous time non linear system $\begin{cases} \dot{x_1} = x_1 - x_1x_1 \\ \dot{x_2} = x_1x_2 - x_2 \end{cases}$ **(04)**

compute the equilibrium points of the system and study the stability of these points using the reduced Lyapunov criterion.

Q.2. B) Answer the following questions: (Any two)

(a) Determine the singular points for $x(x-1)^2y'' - 3y' + 5y = 0$ and classify them. **(03)**

(b) Find the upper bound for $|\vec{f}(x,\vec{y})|$ for $(x,\vec{y})$ in R where $\vec{f}(x,\vec{y}) = (y_2^2 + 1, x + y_1^2)$; $|x| \le 1, y \le 1, \vec{y} \in C_2$. **(03)**

(c) Determine the limit of any solution as t→ ∞ for the system $x' = Ax$, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 1 & -1 & 0 \end{bmatrix}$. **(03)**

Q.3. A) Solve the following equation by power series $x^2y' + \frac{3}{2}xy' + xy = 0$. **(08)**

Q.3. B) Answer the following questions (Any one)

1. Consider the non-homogeneous equation $\frac{d\bar{y}}{dx} = \begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} e^{5x} \\ 4 \end{bmatrix}$ find the unique solution $\bar{\psi}$ of the non-homogeneous equation which satisfies $\bar{\psi}(0) = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$ and having $\overline{\phi_1}(0) = \begin{bmatrix} e^{3x} \\ e^{3x} \end{bmatrix}$, $\overline{\phi_2}(0) = \begin{bmatrix} 3e^{4x} \\ 2e^{4x} \end{bmatrix}$ as pre fundamental set of solutions of corresponding homogeneous equation. **(08)**

2. Let f be a continuous real valued function defined on some interval $R: |x - x_0| \le a, |y - y_0| \le b, (a, b > 0)$ and let $|f(x, y)| \le M$ in the real XY plane. Further suppose that f satisfies a Lipschitz condition with constant K in R. Then the successive approximation $\phi_0 = y_0$, $\phi_{k+1} = y_0 + \int_{x_0}^{x} f(t, \phi_k(t))dt$; $(k = 0,1,2,...)$ converges on the interval $I: |x - x_0| \le \alpha = \min\{a, b/M\}$ to a solution ϕ of the initial value problem $\bar{y}' = \vec{f}(x, \bar{y})$, $y(x_0) = y_0$.

Q.4. A) Find the Green's function for the following B.V.P **(08)**

$y''(x) + y(x) = f(x); y(0) = 0, y'(1) = 0.$

Q.4. B) Answer the following questions (Any two)

(a) Show that any solution of $x' = Ax$ tends to zero as t→ 0 where $A(t) = \begin{bmatrix} -t & \sin t \\ 0 & e^{-t} \end{bmatrix}$. **(03)**

(b) Is the polynomial $r^2 - 2r + 3 = 0$ stable? **(03)**

(c) If $y_1 = x$ is one solution of $x^2y'' + xy' - y = 0$ then find other solution. **(03)**