Seat No: _____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Winter 2017-18 Examination

Semester: 1 **Date: 18/12/2017**

Subject Code: 11206101 **Time: 2:00 pm to 4:30 pm**

Subject Name: Measure Theory **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) **Answer the following questions (Each of 04 marks)** **(08)**

(a) Prove that the interval (a, ∞) is measurable.

(b) State and Prove Countable additivity for Lebesgue Measure.

Q.1. B) **(a) Answer the following questions (Any two)** **(04)**

1. Prove that outer measure of a singleton set is zero.

2. Using the Concept of measure theory prove that [0, 1] is not countable.

(b) If $\{B_k\}_{k=1}^{\infty}$ is an descending collection of measurable sets and $m(B_1) < \infty$ then $m(\bigcap_{k=1}^{\infty} B_k) = \lim_{k\to\infty} m(B_k)$. **(04)**

(c) If f is a measurable function and $f = g$ a.e. then g is measurable. **(04)**

Q.2. A) **(a) Answer the following questions.** **(04)**

1. Define Outer measure. Give an example of a set which is an Uncountable with measure zero.

2. Define Simple function with an example.

(b) Let the function f be defined on a measurable set E then f is measurable if and only if for each open set $\mathcal{O}$, the inverse Image of $\mathcal{O}$ under f is measurable. **(04)**

Q.2. B) **(a) Answer the following questions (Any two)** **(03)**

1. Which one of the following is not true?

a. Outer measure is countable additive
b. Outer measure is translation invariant
c. Outer measure of an interval is its length
d. Outer measure satisfy monotone property

2. If f and g are two measurable real-valued functions defined on the same domain, then

a. $-(f + g)$ are measurable
b. $(fg)^2$ is measurable
c. Both a and b are true
d. Neither a nor b are true

3. Given two non-empty sets A and B, if $m^*(A) = 0$ then

a. $m^*(A \cup B) = m^*(B)$
b. $m^*(A \cup B) \neq m^*(B)$
c. $m^*(A \cup B) = m^*(A)$
d. None of above

(b) Prove that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$. **(03)**

(c) Prove that Characteristic function is a measurable function. **(03)**

Q.3. A) **Answer the following questions:** **(08)**

(a) Let f be a bounded function defined on closed, bounded interval [a, b]. If f is Riemann integrable over [a, b] then prove that it is also Lebesgue integrable over [a, b] and the two integrals are equal.

(b) State and Prove The Bounded Convergence Theorem.

Q.3. B) Answer the following questions (Any two)

(a) For function $f \epsilon L^{\infty}(E)$, define $\|f\|_{\infty}$ = essential supremum of f. Prove that $\|\circ\|_{\infty}$ is norm on $L^{\infty}(E)$. **(04)**

(b) Let f *and* g be bounded measurable functions define on a set of finite measure E . Then for any Scalar α *and* β, Prove that $\int_E \alpha f + \beta g = \alpha \int_E f + \beta \int_E g$. **(04)**

(c) Let E be a measurable set, $1 \leq p < \infty$, If $f, g \epsilon L^{p}(E)$, then so does $f + g$ and $\|f + g\|_p \leq \|f\|_p + \|g\|_p$. **(04)**

Q.4. A) (a) Answer the following questions. **(04)**

1. If $f(x) = \begin{cases} 1 \text{ ; if x is rational} \\ 0 \text{ ; if x is irrational} \end{cases}$ then Show that $R\int_a^b f(x)dx = 0$

2. Let $f(x) = \begin{cases} 1 \text{ } if \text{ } 2 \leq x \leq 4 \\ 4 \text{ } if \text{ } 4 < x < 6 \\ 0 \text{ ; } Otherwise \end{cases}$ then find Lebesgue Integral of function f.

(b) Define a function of Bounded variance and state the necessary and sufficient condition so that a function may be of bounded variation. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Let f be bounded measurable functions define on a set of finite measure E .Suppose A *and* B are disjoints measurable subsets of E. Then Show that $\int_{A \cup B} f = \int_A f + \int_B f$. **(03)**

(b) Let $c \epsilon (a, b)$. If $f \epsilon BV([a, c])$ and $f \epsilon BV([c, b])$ then prove that If $f \epsilon BV([a, b])$ and $T_a^b(f) = T_a^c(f) + T_c^b(f)$. **(03)**

(c) If $f \epsilon BV([a, b])$ then prove that $f'(x)$ exists for almost all x in $[a, b]$. **(03)**