Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Winter 2018-19 Examination

Semester: 3 **Date: 27/10/2018**

Subject Code: 11206203 **Time: 10.30 am to 1.00 pm**

Subject Name: Integral Equations and Calculus of Variation **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions (Each of 04 marks) **(08)**

(a) Find eigen values and corresponding eigen functions of $y(x) = \lambda \int_0^1 e^{x+t} y(t)\,dt$

(b) Find Iterated Kernel of $K(x,t) = \sin(x-2t); \quad 0 \le x,t \le 2\pi$

Q.1. B) Answer the following questions (Any two)

(a) Solve $y(x) = 2x + \lambda \int_0^1 (x+t) y(t)\,dt$, by successive approximation (Neumann Series) method to third order by taking $y_0(x) = 1$. **(04)**

(b) Determine the Resolvent Kernel of $K(x,t) = (1+x)(1-t); \quad a=-1,\ b=1$ **(04)**

(c) Use method of successive approximation to solve $y(x) = 1 + x - \int_0^x y(t)\,dt$, taking $y_0(x) = 1$. **(04)**

Q.2. A) Answer the following questions (Each of 04 marks) **(08)**

(a) Find the extremal of $I[y(x)] = \int_0^{\pi/2} \left(y^2 - y'^2 - 2xy\right) dx; \qquad y(0)=0,\ y(\pi/2)=1$

(b) Show that $y(x) = \left(1+x^2\right)^{-3/2}$ is a solution of the $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)\,dt$

Q.2. B) Answer the following questions (Any two)

(a) Solve $y(x) = e^x + \lambda \int_0^1 2e^{x+t}\, y(t)\,dt$ **(03)**

(b) Convert $y'' + \lambda x y = f(x) \quad ; y(0)=1, y'(0)=0$ into integral equation **(03)**

(c) Convert $y(x) = \frac{x^3}{6} - x + 1 + \int_0^x \left[\sin t - (x-t)\left(e^t + \cos t\right)\right] y(t)\,dt$ into the form of differential equation. **(03)**

Q.3. A) Answer the following questions (Each of 04 marks) **(08)**

(a) With the aid of the Resolvent Kernel, find the solution of the integral equation

$$y(x) = e^{x^2} + \int_0^x e^{x^2 - t^2}\, y(t)\,dt$$

(b) Solve the integral equation $f(x) = \int_a^x \frac{y(t)\,dt}{(\cos t - \cos x)^{1/2}} \qquad 0 \le a < x < b < \pi$

Q.3. B) Answer the following questions (Any two)

(a) Solve $y(x) = 1 + \int_0^1 \left(1 + e^{x+t}\right) y(t)\,dt$ **(04)**

(b) Find Resolvent kernel of $K(x,t) = xe^t$, $a = 0, b = 1$ using Fredholm Determinants. **(04)**

(c) Solve $y(t) = t^2 + \int_0^t y(u)\sin(t-u)\,d$ ιusing convolution theorem **(04)**

Q.4. A) Answer the following questions (Each of 04 marks) **(08)**

(a) Find the extremal of $I[y(x)] = \int_0^{\pi/2} \left(y'^2 - y^2\right) dx;$ $y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$

(b) Show that the $y(x) = \lambda \int_0^1 \left(t\sqrt{x} - x\sqrt{t}\right) y(t)\,dt$ does not have real eigenvalues and eigenfunctions.

Q.4. B) Answer the following questions (Any two)

(a) Find the extremal of $I[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{\left(1 + y'^2\right)}}{x}\,dx$ **(03)**

(b) Find the length of arc between the points 0 and 1 for $y(x) = x$ **(03)**

(c) Solve the singular integral equation $x = \int_0^x \frac{1}{(x-t)^{1/2}} y(t)\,dt$ **(03)**