Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Winter 2018-19 Examination

Semester: 3 **Date: 25/10/2018**
Subject Code: 11206202 **Time: 10.30 am to 1.00 pm**
Subject Name: Mathematical Modeling **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions (Each of 04 marks) **(08)**

(a) Derive the mathematical model to the problem of pollution in a lake. Assume that the lake has a constant volume V, and that is continuously well mixed so that the pollution is uniform throughout. Let C(t) be the concentration of the pollutant in the lake at time t. Let F be the rate at which water flow in-out of the lake and c_{in} be the concentration of the pollutant in the flow entering the lake. Solve the model with initial condition C(0) = c_0 .

(b) For the lake pollution model find (i) all equilibrium points, (ii) stability of equilibrium points (iii) how long it will take for the lake's pollution level to reach 5% of its initial level if only fresh water flows into the lake.

Q.1. B) Answer the following questions (Any two)

(a) The Rate of change of atmospheric pressure p with respect to height is assumed proportional to p. If p = 14.7 psi at h = 0 feet and p = 7.35 psi at h = 17,500 feet, what is p at h= 10,000 feet? **(04)**

(b) Write down the MATLAB program to solve the discrete logistic equation $x_{n+1} = x_n + rx_n(1 - \frac{x_n}{k})$ with $r = 0.2, k = 1000\ and\ x(0) = 100.$ **(04)**

(c) Formulate and solve mathematical modeling for radioactivity on the interval [0, t] with N (0) = N_0. **(04)**

Q.2. A) Answer the following questions.

(a) A large tank contains 1000 liters of salt water. Initially s_0 kg of salt dissolved. Salt water flows in to the tank at the rate of 10 liter per minute and the concentration $c_{in}(t)$ of this incoming water-salt mixture varies with time. Assume that the solution in the tank is thoroughly mixed and that the salt solution flows out at the same rate at which flows in, that is, the volume of the water salt in the tank remains constant. Find a differential equation for the amount of salt in the tank at any time t. Also solve it. **(04)**

(b) Let $S(t)$ and $I(t)$ be the number of susceptible and infected persons respectively at time t. Let $S(t) + I(t) = n + 1, S(0) = n, I(0) = 1$, Construct mathematical model for the rates of change of susceptible and invectives. Also solve it. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) If τ denote the half-life of the radioactive substance then find the value of the constant k in terms of τ for radioactivity mathematical model. **(03)**

(b) Find the equilibrium solutions for the model $\frac{dx}{dt} = xr(1 - \frac{x}{k})$. Also check stabilities. **(03)**

(c) Explain (i)the distinction between temperature and Heat. (ii)specific Heat **(03)**

Q.3. A) Answer the following questions (Each of 04 marks) **(08)**

(a) Let $x(t), y(t)$ be the population of the prey and predator species at time t. Assume that the presence of both predators and prey is beneficial to growth of predator species and is harmful to growth of prey species. More specifically the predator species increases and the prey species decreases at rates proportional to the product of two population. Formulate mathematical model and solve it.

(b) Write down the MATLAB program to solve $\frac{dy}{dx} = sinx + cosy\,; y(0) = 5, h = 0.1$ by Range-Kutta 4^{th} order method.

Q.3. B) Answer the following questions (Any two)

(a) A population, initially consisting of 1000 mice, has a per capita birth rate of 0.8 mice per month (per mouse) and a per capita death rate of 2 mice per month (per mouse). Also 20 mice traps are set each week and they are filled. Explain the rate of change in the number of mice as differential equations and solve it. **(04)**

(b) A hot cup of coffee is at a temperature of 60℃ , and after 10 minutes the coffee has cooled to 50℃ .The room temperature is 20℃ then find how long it take to cool down to 40℃ . **(04)**

(c) Suppose that we have a standard home hot water tank, which holds 250 liters of water and is cylindrical with a height of 1.444 m and a diameter of 0.564m. The water is heated by heating element immersed in the water supplies heat at a constant rate of 3600 (watts) (q=3600w=3600 J /Sec). Assume that mass of the water to be 250 kg (as a litre of water has the mass of approximately 1 kg) with the surface area of the tank approximately 3.06m^2. The specific heat of water is c = 4200 J / kg ℃ and heat transfer coefficient is h = 12Wm^{-2}°C^{-1}. Given that the temperature of surrounding is$u_s = 15$℃. Initially the water in the tank has the same temperature as its environment ($u_s = u_0$). How long it would take the water to reach a temperature of 60℃ ? **(04)**

Q.4. A) Answer the following questions.

(a) Derive the model of Heat Conduction through a wall as differential equation and solve it. **(04)**

(b) Obtain a differential equation for the equilibrium temperature inside the annular shell. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Explain Fourier's Law of Heat Conduction. **(03)**

(b) Explain technique of mathematical modeling. **(03)**

(c) Use Euler's method, with step size $h = 0.1$, to find $y(0.2)$ for the differential equation $\frac{dy}{dt} = y^2 + t,\ y(0) = 1.$ **(03)**