Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Winter 2018-19 Regular Examination

Semester: 3 **Date: 23/10/2018**
Subject Code: 11206201 **Time: 10.30 am to 1.00 pm**
Subject Name: Functional Analysis **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Let f be a bounded linear functional on a subspace Z of a normed space X. then there exists a bounded linear functional $\tilde{f}$ on X which is an extension of f to X and has the same norm $||\tilde{f}||_x = ||f||_z$ **(08)**

where $||\tilde{f}||_x = \sup_{\substack{x \epsilon X \\ ||x||=1}} |\tilde{f}(x)|$, $||f||_z = \sup_{\substack{x \epsilon Z \\ ||x||=1}} |f(x)|$.

Q.1. B) Answer the following questions (Any two)

(a) True or False with Justification (Each of 02 marks) **(04)**

1. Every finite dimensional subspace Y of a normed space X is closed in X.
2. Every reflexive normed space X is complete.

(b) Let $X = (X, d)$ be a metric space then **(04)**

i. A convergent sequence in X is bounded & its limit is unique.

ii. If $x_n \to x$ and $y_n \to y$ in X then $d(x_n, y_n) \to d(x, y)$.

(c) In a finite dimensional normed space X, any subset $M \subset X$ is compact if and only if M is closed and bounded. **(04)**

Q.2. A) Answer the following questions.

(a) Fill in the blanks. (Each of 02 marks) **(04)**

1. __________ is not separable.

a) $\mathbb{R}$ **b)** $\mathbb{C}$ **c)** l^∞ **d)** l^p

2. _________ is not reflexive.

a) C[a,b] **b)** $l^p[a,b]$ **c)** l^∞ **d)** $l^p, 1 < p < +\infty$

(b) If Y is Banach space, then $B(X, Y)$ is Banach space. **(04)**

Q.2. B) Answer the following questions (Any two)

(a)Do as directed:(Each of 01 marks) **(03)**

1. Does every bounded linear operator defined on a subspace have a norm preserving extension?

2. If f is a non-zero linear functional on an infinite dimensional linear space X, does there exist a norm on X such that f is discontinuous?

3. In which space, every convergent sequence is a Cauchy sequence?

(b) A subspace M of a complete metric space X is itself complete if and only if the set M is closed in X. **(03)**

(c) A mapping T of a metric space X into a metric space Y is continuous if and only if the inverse image of any open subset of Y is an open subset of X **(03)**

Q.3. A) The dual space of l^p; here $1 < p < +\infty$ and q is the conjugate of p, that is $\frac{1}{p} + \frac{1}{q} = 1$. **(08)**

Q.3. B) Answer the following questions (Any two) **(08)**

(a) Every finite dimensional subspace Y of a normed space X is complete. In particular, every finite dimensional normed space is complete.

(b) Let Y be any closed subspace of a Hilbert space H then $H = Y \oplus Z; Z = Y^{\perp}$.

(c) Let $T: X \to X$ be a continuous mapping on a complete metric space $X = (X, d)$, and suppose that T^m is a contraction on X for some positive integer m. then T has a unique fixed point.

Q.4. A) The Hilbert-adjoint operator T^* of T in exists, is unique and is a bounded linear operator with norm $||T^*|| = ||T||$. **(08)**

Q.4. B) Answer the following questions (Any two)

(a)Do as directed (Each of 01 marks) **(03)**

1. The dual space of l^1 is ________.
2. "All normed spaces are inner product space." is this statement true?
3. Define Banach space.

(b) Let M be a nonempty subset of a metric space(X, d) and $\tilde{M}$ its closure as defined then $x \in \overline{M}$ iff there is a sequence (x_n) in M such that $x_n \to x$. **(03)**

(c) The product of two bounded self-adjoint linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute, $ST = TS$. **(03)**