Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Winter 2018-19 Examination

Semester: 1 **Date: 10/12/2018**
Subject Code: 11206105 **Time: 10:30 am to 01:00 pm**
Subject Name: Number Theory **Total Marks: 60**

Instructions:
1. All questions are compulsory
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions (4x2) (Each of 04 marks) **(08)**
(a) State and prove Fundamental theorem of Divisibility.
(b) If $(a, b) = d$ then prove that $\exists\ x, y \in Z$ such that $ax + by = d$.

Q.1. B) Answer the following questions (Any two)
(a) Answer the following. (Each of 02 marks) **(04)**
1.Prove that $(a - s)|\ (ab + st)$ then $(a - s)|\ (at + sb)$
2.If $a|c$ and $b|c$ and $(a, b) = 1$ then prove that $ab|c$
(b) State & prove Chinese remainder theorem. **(04)**
(c) Prove that common multiple of two non-zero integers is also a multiple of their LCM. **(04)**

Q.2. A) Answer the following questions.
(a) Answer the following. (Each of 02 marks) **(04)**
1) Show that congruence is an equivalent relation.
2) If $a \equiv b(\mathrm{mod}\, n)$ *and* $c \equiv d(\mathrm{mod}\, n)$ then show that $a + c \equiv b + d(modn)$
(b) If a is square number then show that $S(a)$ is an odd integer. (S(a) is sum of Divisors) **(04)**

Q.2. B) Answer the following questions (Any two)
(a) Multiple choice questions. (Each of 01 marks) **(03)**
1) Define Greatest integer Function.
2) $lcm[a\ , 0] =$_______
a) 0 b) 1 c) a d) does not exists
3) If $a(a > 2)$ is a composite number then T(a) is always greater than 2*a.(true/False)
(b) Prove that $[x]+[y]\le[x+y]\le[x]+[y]+1$ (here [x] is greatest integer function.) **(03)**
(c) Show that Mobious function is multiplicative function, when $(a, b) = 1$ **(03)**

Q.3. A) Answer the following questions (4x2) (Each of 04 marks) **(08)**
(a) If $(a, b) = 1$ then prove that $(a, bc) = (a, c)$
(b) Prove that $a \equiv b(\mathrm{mod}\, n)$ $iff\ a\ and\ b$ have same non negative remainders when divided by n.

Q.3. B) Answer the following questions (Any two)
(a) Short note. (Each of 02 marks) **(04)**
1) If the value of T(a) = 2 then find the value of P(a) and S(a).
2) If $a_1,a_2,a_3,a_4,\ldots..a_m$ is CRS modulo m and $(a, m) = 1$ then prove that $aa_1 + m, aa_2 + m, aa_3 + m, \ldots, aa_m + m$ forms CRS modulo m, where b is any integer.
(b) State and prove Fermat's theorem. **(04)**
(c) Solve $12x + 15 \equiv 0(mod\ 45)$ **(04)**

Q.4. A) Answer the following questions.
(a) Short Questions (Each of 02 marks) **(04)**
1) Let $p \neq 2$ be a prime number and $a,$ b be the integers such that p does not divide $a\ and\ b$ but $p|(a - b)$ then prove that $\left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$
2) Prove that there is no prime p of the form $4k + 3$ which is expressed as a sum of two squares.
(b) State and prove unique factorization theorem. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Multiple choice questions. (Each of 01 marks) **(03)**

1) Which of the following is not an algebraic number?

a) 0 b) ½ c) -5 d)none of these

2) Which of the following is an prime ideal?

a) $2\mathbb{Z}$ b) $4\mathbb{Z}$ c)$6\mathbb{Z}$ d) $8\mathbb{Z}$

3) $x \equiv 2 (\bmod 7)$ is a quadratic residue modulo n?(True/False)

(b) Find positive integer solution for $7x + 19y = 213$ **(03)**

(c) Prove that positive integer solution of $x^{-1} + y^{-1} = z^{-1}$ (where $(x, y, z) = 1$) has and must have the form $x = a(a + b)$, $y = b(a + b)$ $z = ab$ where $a, b > 0$ and $(a, b) = 1$ **(03)**