Seat No:_____________ Enrollment No:_____________

# PARUL UNIVERSITY

## FACULTY OF APPLIED SCIENCE

## M.Sc., Winter 2018-19 Examination

**Semester: 1** **Date: 07/12/2018**
**Subject Code:11206104** **Time: 10:30am to 01:00pm**
**Subject Name: Topology** **Total Marks: 60**

**Instructions:**
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

**Q.1. A) Attempt any one** **(08)**

(a)If $X$ and $Y$ be two topological spaces and $f: X \to Y$ is a continuous map, then prove the following:

(i) For every subset $A$ of $X$, $f(\bar{A}) \subset \overline{f(A)}$.

(ii) For every $x \in X$ and every neighborhood $V$ of $f(x)$ there is a neighborhood $U$ of $x$ such that$f(U) \subset V$.

(b) Prove that the product of finitely many compact topological spaces is compact.

**Q.1. B) Answer the following questions (Any two)**

(a) Do as directed: **(04)**

1. Let X and Y are topological spaces. Describe under which condition a function f : X $\to$ Y is said to be continuous?

2. State what it means for a subset of a topological space to be path connected.

(b) Prove that a compact subset of a Hausdroff space is closed. **(04)**

(c) Prove that every metric space is $T_2$ space. **(04)**

**Q.2. A) Answer the following questions.**

(a) State whether the following statement are true or false with justification **(04)**

1. All Hausdroff space with countable many points are compact.
2. Finite topological spaces are always connected.

(b) Prove that continuous image of a connected space is connected. **(04)**

**Q.2. B) Answer the following questions (Any two)**

(a) Short note/ Multiple choice questions. (Each of 01 marks) **(03)**

1. A projection map $\pi_1: X \times Y \to X$ is always________.

(i) Continuous and one-one (ii) Continuous and onto
(iii) Neither continuous nor onto (iv) Continuous but not onto.

2. Let $Y = [-1,1] \cup (2,3)$ be a subspace topology of real line with usual topology on $\mathbb{R}$. The set $[-1,1]$ in $Y$ is

(i) open but not closed (ii) closed but not open
(iii) neither closed nor open (iv) both open and closed

3. If $\tau_1$ and $\tau_2$ are two typologies on non-empty set X, then ……………… is topological space.

(i)$\tau_1 \cup \tau_2$ (ii) $\tau_1 \cap \tau_2$ (iii) $\tau_1 \setminus \tau_2$ (iv) $\tau_2 \cup \tau_1$

(b) Prove that a closed subset of a compact space is compact. **(03)**

(c) Show that every closed subspace A of a normal space X is normal. **(03)**

**Q.3. A)** Do as directed: **(08)**

(a)State and Prove Lindelof's Theorem.

(b)Prove that separable metric space is second countable.

**Q.3. B) Answer the following questions (Any two)**

(a) Do as directed: **(04)**

1. Let $A = \{1,2,3,4,5\}$ with discrete topology $P(A)$. Is it separable? What is the basis for the given topology?

2. If $(X, \tau)$ is a topological space, where $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a, c\}, \{b, d\}\}$, what is the set of limit points of $X$ ? Which points are isolated?

(b) Show that if U is open in X and A is closed in X , then U−A is open in X , and A−U is closed in X. **(04)**

(c) Let f be a map from topological space X on to a set A. Prove that there exists exactly one topology T on A relatively to which f is a quotient map. **(04)**

**Q.4. A) Answer the following questions.**

(a) Do as directed: **(04)**

1. Let(X, T) be a topological space. Let A be a subset of X. Define the closure of A.
2. Define when a topological space X is locally compact.

(b) Show that the components of X are connected disjoint subspaces of X such that each non-empty connected subspaces of X intersects only one of them. **(04)**

**Q.4. B) Answer the following questions (Any two)**

(a) Prove that a metric space with Bolzano Wierstrass property is a sequentially compact metric space. **(03)**

(b) Prove that U is open if and only if U = int(U). **(03)**

(c) A topological space X is $\tau_{1,}$ iff all singletons are closed. **(03)**