Seat No: ______________ **Enrollment No: ______________**

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. (Mathematics) Winter2018-19 Examination

Semester: 1 **Date: 05/12/2018**
Subject Code: 11206103 **Time: 10:30 am to 01:00 pm**
Subject Name: Advanced Numerical Analysis **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following (Each of 04 marks)

(a) Find a root of the equation $x^3-4x-9=0$ using bisection method correct to two decimal places; where root lies between $x=2$ and $x=3$. **(04)**

(b) Solve $\frac{d^2y}{dx^2}+4y=\cos x$ with $y(0)=y\left(\frac{\pi}{4}\right)=0$ and step size $h=\frac{\pi}{20}$ by finite difference method. **(04)**

Q.1. B) Answer the following questions (Any two)

(a) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2$ **(04)**

(b) Prove that the Newton- Raphson method for finding the root of the equation $f(x)=0$ has a quadratic convergence. **(04)**

(c) Using Milne's Predictor-corrector method, Obtain the solution of $\frac{dy}{dx}=x-y^2$ at $x=0.8$,given that $y(0)=0.0000,\ y(0.2)=0.0200,\ y(0.4)=0.0795,\ y(0.6)=0.1762$ **(04)**

Q.2. A) Answer the following questions.

(a) Do as directed: (Each of 02 marks) **(04)**

1. Evaluate $\frac{1}{\sqrt{14}}$ correct to four decimal places by Newton's iteration method.
2. Find the local minimum of the function $f(x)=x^2-4x+y^2-y-xy$ by using second partial derivative test.

(b) Determine the largest eigen value and corresponding eigen vector by Power method of the matrix $A=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$ considering the initial eigen vector as $X_0=\begin{bmatrix}1\\0\\0\end{bmatrix}$. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) **Choose the correct answer for the following multiple choice questions. (Each of 01 marks)** **(03)**

1. If $f(x)$ is continuous on the interval , $I=[a,b]$ and if $f'(x)$ is defined for all $x\in(a,b)$, except possibly at $x=p$, where $p\in(a,b)$ then,

   a. If $f'(x)<0$ on (a,p) and $f'(x)>0$ on (p,b), then $f(p)$ is a local minimum.

   b. If $f'(x)>0$ on (a,p) and $f'(x)<0$ on (p,b), then $f(p)$ is a local minimum.

   c. If $f'(x)>0$ on (a,p) and $f'(x)>0$ on (p,b), then $f(p)$ is a local minimum.

   d. If $f'(x)<0$ on (a,p) and $f'(x)<0$ on (p,b), then $f(p)$ is a local minimum.

2. The error in Simpson's $\frac{3^{th}}{8}$ rule is of order

   a. 5 b. 4 c. 7 d. 3

3. The partial differential equation (PDE) $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x \partial y}+4\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ is a/an

a. Elliptic PDE b. Hyperbolic PDE c. Parabolic PDE d. Laplace equation

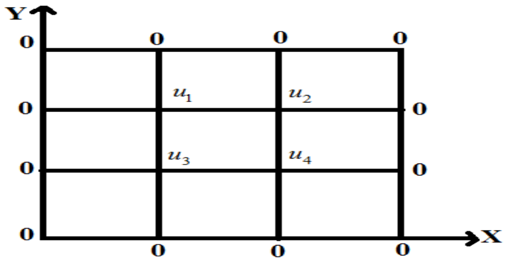
(b) **Answer the following:** **(03)**

1. State the iterative formula for finding solution of $\frac{dy}{dx}=f(x,y),\ y(x_0)=y_0$ by Heun's method.

2. If $AX=B$ be the system of linear equations with n number of variables, then state the condition in terms of rank of a matrix A and $[A\,|\,B]$, which represents a unique solution.

(c) Solve the system of non-linear equations by Newton-Raphson method $x^2+y=11,\ y^2+x=7$ with the initial approximation $x_0=3.5$ and $y_0=-1.8$. Approximate the root for only one iteration. **(03)**

Q.3. A) Solve the equation $\nabla^2 u=-10\left(x^2+y^2+10\right)$ over the square with the sides $x=0, y=0, x=3, y=3$ with $u=0$ on the boundary and mesh length equals to 1 as shown in below figure by finite difference method. **(08)**



Q.3. B) Answer the following questions (Any two)

(a) Derive optimum size for central difference formula with $O(h)=4$, where h is the step size. **(04)**

(b) Reduce the matrix $\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix}$ to the tri-diagonal form, using House-Holder's method. **(04)**

(c) Using Shooting method, solve the boundary value problem: $y''(x)=y(x),\ y(0)=0$ and $y(1)=1.17$. **(04)**

Q.4. A) Use Nelder-Mead algorithm to find the minimum of $f(x,y)=x^3+y^3-3x-3y+5$. Start with three vertices $V_1=(0,0),\ V_2=(1.2,0.0),\ V_3=(0.0,0.8)$. Compute four iterations. **(08)**

Q.4. B) Answer the following questions (Any two)

(a) Find by Taylor's series method, the values of y at $x=0.2$ to five decimal places from $\frac{dy}{dx}=x^2y-1,\ y(0)=1$. **(03)**

(b) For the given values: **(03)**

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=1.1$

(c) Evaluate the integral $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates using Simpson's $\frac{1}{3}^{rd}$ rule. **(03)**