Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Winter 2018-19 Examination

Semester: 1 **Date: 03/12/2018**
Subject Code: 11206102 **Time: 10:30am to 01:00pm**
Subject Name: Theory of Ordinary Differential Equation **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) For non-homogeneous linear systems $\frac{d\bar{y}}{dx} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}\bar{y} + \begin{bmatrix} 4e^{3x} \\ -e^{3x} \end{bmatrix}$ find the fundamental matrix corresponding to the homogeneous system and also find solution of the given non-homogeneous system $x_0 = 0$. **(08)**

Q.1. B) Answer the following questions :(Any two)

(a) Do as directed: **(04)**

1. Justify the B.V.P $x'' + |x| = 0; 0 < t < \pi$ with boundary condition $x(0) = x(\pi) = 0$ is not linear.

2. Let $f(x,y) = y^{\frac{2}{3}}$ on $R: |x| \le 1, y \le 1$ check whether f satisfies Lipschitz condition on R.

(b) Solve the equation $y^2 y'' = y'$. **(04)**

(c) Find a solution $\vec{\phi}$ of the given system $y_1' = y_1;\ y_2' = y_1 + y_2$ which satisfies $\vec{\phi}(0) = (1,2)$. **(04)**

Q.2. A) Answer the following questions:

(a) State whether the following statement are true or false with justification **(04)**

1. For $\bar{y}' = \vec{f}(x,\vec{y})$ with $\vec{f}(x,\vec{y}) = (y_2, -y_1)$ and $\vec{y}(0) = (0,1)$, the successive approximation $\emptyset_2$ is given by $\emptyset_2(x) = \left(x, \frac{1-x^2}{2}\right)$.

2. $t = 0$ is an ordinary point for the differential equation $t^2 x'' + tx + t^2 x = 0$,

(b) Find the indicial polynomials for $t^2 x'' + t(3-2t)x' + (1-3t+t^2) = 0$. **(04)**

Q.2. B) Answer the following questions: (Any two)

(a) If $y_1(x)$ and $y_2(x)$ are two solutions of the equation $y'' + P(x)y' + Q(x) = 0$ on [a,b] then prove that their Wronskian $W = (y_1, y_2)$ is identically equal zero or never zero on [a,b]. **(03)**

(b) Find the basis for the solution of the equation $x^2 y'' + xy' + y = 0;\ x \ne 0$. **(03)**

(c) Show that $x = \infty$ is a regular singular point of $x^2 y'' + 4xy' + 2y = 0$. **(03)**

Q.3. A) Verify that origin is regular singular point and calculate two independent Frobenius Series solutions for the equation $2xy'' + (x+1)y' + 3y = 0$. **(08)**

Q.3. B) Answer the following questions (Any one)

1. Consider the IVP $y' = 3y + 1, y(0) = 2.$ **(08)**

a)Show that all successive approximation $\emptyset_0, \emptyset_1, \emptyset_2, \dots\dots$ exist for all real x.

b) Compute the first $\emptyset_0, \emptyset_1, \emptyset_2, \emptyset_3$ to be the solution.

c) Compute the solution of the IVP

d) Compute the result of (b) and (c)

2. Let f be a continuous real valued function defined on some interval $R : |x - x_0| \le a, |y - y_0| \le b, (a, b > 0)$ and let $|f(x, y)| \le M$ in the real XY plane. Further suppose that f satisfies a Lipschitz condition with constant K in R. Then the successive approximation $\phi_0 = y_0$, $\phi_{k+1} = y_0 + \int_{x_0}^{x} f(t, \phi_k(t))dt$;$(k = 0,1,2,\dots)$ converges on the interval $I : |x - x_0| \le \alpha = \min\{a, b/M\}$ to a solution ϕ of the initial value problem $\bar{y}' = \vec{f}(x, \vec{y})$, $y(x_0) = y_0$.

Q.4. A) Find the Green's function for the following B.V.P $y'' = f(x), y(0) = y(1) = 0.$ Hence solve $y''(x) = x^2$ subject to the same B.C **(08)**

Q.4. B) Answer the following questions (Any two)

(a) Prove that $[x^P J_P(x)]' = x^P J_{P-1}(x)$ **(03)**

(b) Is the polynomial $r^2 + 2r + 3 = 0$ stable? **(03)**

(c) Verify that $y_1 = x^2$ is one solution of $x^2y'' + xy' - 4y = 0$ then find y_2 and general solution. **(03)**