Seat No: ______________ **Enrollment No: ______________**

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Winter 2019-20 Examination

Semester: 3 **Date: 28/11/2019**
Subject Code: 11206202 **Time: 02:00 pm to 04:30 pm**
Subject Name: Mathematical Modelling **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Write a short note on Mathematical Modeling. Also write merits and demerits of mathematical modeling. **(08)**

Q.1. B) Answer the following questions (Any two)

(a) Write assumption for prey- predator model. Also write the word equation. **(04)**

(b) Use Runge-Kutta method, with step size h=0.2, find $y(0.2)$ for differential equation **(04)**
$\frac{dy}{dt} = y^2 - t;^2 \quad y(0) = 1.$

(c) Use Euler's method, with step size h=0.1, find $y(0.2)$ for differential equation **(04)**
$\frac{dy}{dx} = x + y \quad y(0) = 1.$

Q.2. A) Determine the rate at which a drug, antihistamine or decongestant moves between two compartment in the body, the GI-tract and bloodstream, when a patient takes a single pills. Let $x(t)$ is level of the drug in the GI-tract and $y(t)$ the level in the bloodstream at time t. **(08)**
(i) Develop a mathematical model for cold pills in the different compartment.
(ii) Find the expression for $x(t)$ and $y(t)$ which satisfy this pair of differential equations, when $k_1 \neq k_2$.

Q.2. B) Answer the following questions (Any one)

(a) Develop a mathematical modeling of radioactive decay of an element. Find the amount of radioactive element at time t. Derive the expression for the half life time of a radioactive element. If an archaeologist uncovers a sea shell which contains 60% of the 14_C of a living shell, how old do you estimate that shell, and thus that site, to be? (You may assume the half-life of 14_C to be 5368 years). **(06)**

(b) Model for the spread of technology are very similar to the logistic model for population growth. Let N(t) be number of ranchers who have adopted an improved pasture technology in India. Then N(t) satisfies the differential equation **(06)**

$$\frac{dN}{dt} = aN\left(1 - \frac{N}{N^*}\right)$$

Where N* is the total population of ranchers. According to banks, N*= 17015, a=0.490, N_0=141. Determine how long it takes for the improved pasture technology to spread to 80% of the population.

Q.3. A) Answer any one

(a) Write Newton's law of cooling. Consider a coffee is at temperature 60^o initially and surrounding temperature is 25^o. After 10 minutes temperature of coffee is 35^o. How long it takes to cool down at 30^o temperature. **(08)**

(b) Develop a mathematical model for growth of humans in a city having ~ as birth rate and as a death rate $\frac{dx}{dt} = rx$; where r=~ - , Suppose initially population of humans 1 million and birth rate is 2 per day and death rate is 1 per day. Find the population of human after at any time t. When will be the population is 1.5 times the original population. **(08)**

Q.3. B) Answer the following questions (Any two)

(a) Explain Compartment model with example. **(04)**

(b) For the differential equation $\frac{dX}{dt} = -XY;\ \frac{dY}{dt} = -2Y$. Find the relation for X and Y, using chain rule. **(04)**

(c) Discuss the equilibrium point and phase plane diagram for $\frac{dX}{dt} = -Y, \frac{dY}{dt} = -X$. **(04)**

Q.4. A) Answer the following questions.

(a) Short note/ Brief note (2x2)/ Fill in the blanks. (Each of 02 marks) **(04)**

1. Find the equilibrium point for the system $x' = x - 5y; \quad y' = x - y$.
2. State Newton's law of cooling

(b) Consider there is war between blue and red army, state assumptions for red and blue army. Write a word equation and hence differential equations for red army and blue army. **(04)**

Q.4. B) Answer any one of the following:

(a) Consider the concentration of pollution in lake is $C(t)$ and initially concentration of pollutant in lake is $C(0)=C_0$. The equation of concentration of pollution is given by $\frac{dC}{dt} = \frac{F}{V}C_{in} - \frac{F}{V}C.$ where F is rate at which flows out and it is constant and V is constant volume of lake, C_{in} is concentration of pollutant in the flow entering the lake. **(06)**

(i) Find the concentration of pollutant at any time t.

(ii) What will be concentration of pollutant as $t \to \infty$.

(iii) How long will it take for the lake pollution level to reach 5% of its initial level, if only fresh water flows into the lake.

(iv) Find the equilibrium point for this system.

(b) Suppose a population can be modelled using the differential equation $\frac{dx}{dt} = 0.2x - 0.001x^2; \quad x(0) = 100.$ Suppose time step is 1 month, Find the predicted population after two months. **(06)**