Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Winter 2019-20 Examination

Semester: 3 **Date: 26/11/2019**
Subject Code: 11206201 **Time: 2:00pm to 4:30pm**
Subject Name: Functional Analysis **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) If Y is Banach space, then $B(X,Y)$ is Banach space. **(08)**

Q.1. B) Answer the following questions (Any two)

(a) Let $X = (X, d)$ be a metric space then **(04)**
  a. A convergent sequence in X is bounded & its limit is unique.
  b. If $x_n \to x$ and $y_n \to y$ in X then $d(x_n, y_n) \to d(x, y)$.

(b) Every finite dimensional subspace Y of a normed space X is complete. In particular, every finite dimensional normed space is complete. **(04)**

(c) If a normed space X has the property that the closed unit ball $M = \{x / ||x|| \leq 1\}$ is compact ,then X is finite dimensional. **(04)**

Q.2. A) Answer the following questions.

(a) A subspace M of a complete metric space X is itself complete if and only if the set M is closed in X. **(04)**

(b) Let Y be any closed subspace of a Hilbert space H. then $H = Y \oplus Z$; $Z = Y^{\perp}$. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) The product of two bounded self-adjoint linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute, $ST = TS$. **(03)**

(b) Let X be an $n-$ dimensional vector space. Then any proper subspace Y of X has dimensional less than n. **(03)**

(c) Let X be a normed space and let $x_0 \neq 0$ be an element of X. Then there exists a bounded linear functional $\bar{f}$ on X such that $||\bar{f}|| = 1$, $\bar{f}(x_0) = ||x_0||$. **(03)**

Q.3. A) An inner product and the corresponding norm satisfy the Schwarz inequality and the triangle inequality as follows. **(08)**
  a. We have $|\langle x, y\rangle| \leq |x||y|$ where the equality sign holds if and only if $\{x, y\}$ is a linearly dependent set.
  b. That norm also satisfies $||x + y|| \leq ||x|| + ||y||$ where the equality sign holds if and only if $y = 0$ or $x = cy$ (c real and ≥ 0).

Q.3. B) Answer the following questions (Any two)

(a) A mapping $T: X \to Y$ of a metric space (X, d) into a metric space $(Y, \bar{d})$ is continuous at a point $x_0 \in X$ if and only if $x_n \to x_0 \Rightarrow Tx_n \to Tx_0$. **(04)**

(b) In a finite dimensional normed space X, any subset $M \subset X$ is compact if and only if M is closed and bounded. **(04)**

(c) Let $T: \mathcal{D}(T) \to Y$ be a bounded linear operator with domain $\mathcal{D}(T) \subset X$, where X and Y are normed spaces. Then: **(04)**
  a. If $\mathcal{D}(T)$ is closed subset of X, then T is closed.
  b. If T is closed and Y is complete, then $\mathcal{D}(T)$ is closed subset of X.

Q.4. A) Consider a metric space $X = (X, d)$, where $X \neq \phi$. Suppose that X is complete and let $T: X \to X$ be a contraction on X. Then T has precisely one fixed point. **(08)**

Q.4. B) Answer the following questions (Any two)

(a) Every convergent sequence in a metric space is a Cauchy sequence. **(03)**

(b) If $\langle v_1, w \rangle = \langle v_2, w \rangle$ for all w in an inner product space X, then $v_1 = v_2$. In particular, $\langle v_1, w \rangle = 0$ for all $w \in W$ implies $v_1 = 0$. **(03)**

(c) Let X & Y be metric spaces and $T : X \to Y$ a continuous mapping .Then the image of a compact subset M of X under T is compact. **(03)**