PARUL UNIVERSITY
FACULTY OF APPLIED SCIENCE
M.Sc./IMSC Winter 2019-20 Examination

Semester: 1/7   Date: 02/12/2019
Subject Code: 11206107   Time: 10:30 am to 01:00 pm
Subject Name: Transform Theory   Total Marks: 60

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following

(a) Find the Fourier Transform e^{-ax^2}. Also prove that $F(k) = F(e^{-ax^2}) = \dfrac{1}{\sqrt{2a}} e^{\frac{-k^2}{4a}}, a > 0$. Also **(08)**

draw graph.

Q.1. B) Answer the following questions (Any two)

(a) Prove that "The Fourier Transform of a good function is a good function". **(04)**

(b) Prove that $F(f(x-a)) = e^{-ika} F(f(x))$. **(04)**

(c) Find the convolution of $f(x) = \cos x \ \& \ g(x) = e^{-a|x|}$. **(04)**

Q.2. A) Answer the following questions.

(a) Find Fourier Transform of $f(x) = \begin{cases} 0, & x < 0 \\ 1/2, & x = 0 \\ e^{-x}, & x > 0 \end{cases}$. Prove that **(08)**

$$\int_0^\infty \frac{\cos kx + k \sin kx}{1 + k^2} dk = \begin{cases} 0, & x < 0 \\ f/2, & x = 0 \\ fe^{-x}, & x > 0 \end{cases}.$$

Q.2. B) Answer the following questions (Any two)

(a) Fill in the blanks. (Each of 01 marks) **(03)**
     1. $F(\mathsf{u}(t-a)) = $ _____.

     2. $F(e^{-a|t|}) = $ _____.

     3. $F^{-1}\left(\dfrac{1}{a+is}\right) = $ _____ .

(b) Solve by Fourier Transform $\dfrac{dy}{dt} - 4y = e^{-4t} H(t)$, where H(t) is Heaviside function. **(03)**

(c) Find the Fourier sine and cosine Transform of $f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x > a \end{cases}$. **(03)**

Q.3. A) Answer the following questions.

(a) Evaluate $(i) L^{-1}\left(\dfrac{p+2}{(p-1)(p^2+4)}\right) (ii) L^{-1}\left(\dfrac{p-3}{p^2+4}\right)$ **(08)**

Q.3. B) Answer the following questions (Any two)

(a) Evaluate $L(\cos(2t-3))$. **(04)**

(b) Using Laplace transform of derivatives find $L(te^{2t})$. **(04)**

(c) Find the Laplace Transform of the periodic waveform $f(t) = \dfrac{kt}{T}, 0 \le t \le T$. [sawtooth wave]. **(04)**

Q.4. A) Answer the following questions.

(a) Using residue technique, find $Z^{-1}\left\{\dfrac{z^2}{(z-a)(z-b)}\right\}$. **(04)**

(b) Using convolution theorem, evaluate $Z^{-1}\left\{\dfrac{z^2}{(z-a)(z-b)}\right\}$. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Find $Z(n+1)$. **(03)**

(b) Find the Z-transform of the following $f_n = 1$. **(03)**

(c) If $Z(f_n) = F(z)$ then prove that $Z(n^k) = -z\dfrac{d(z(n^{k-1}))}{dz}$. **(03)**