Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

B.Sc. Winter 2018-19 Examination

Semester: 1 **Date: 22/12/2018**
Subject Code: 11106101 **Time: 10:30 am to 1:00 pm**
Subject Name: Mathematics-I **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions (Each of 04 marks) **(08)**

(a)If u and v are differentiable functions of x then, show that $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

(b)Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^2-y^2}{\sqrt{x^2+y^2}} & ;(x,y) \neq (0,0) \\ 0 & ;\ (x,y) = (0,0) \end{cases}$ at $(0,0)$.

Q.1. B) Answer the following questions (Any two)

(a) Do as directed (Each of 02 marks) **(04)**
1. Write the domain and range of the function $f(x) = \cos x$.
2. Evaluate $\frac{dy}{dx}$ if $y = x^5 - \log x + 7$.

(b) Trace the curve $r = 2(1 + \cos\theta)$. **(04)**

(c) If $y = \cos(m sin^{-1}x)$, show that **(04)**
$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

Q.2. A) Answer the following questions.

(a) Do as directed (Each of 02 marks) **(04)**
1.Write the statement of Rolle's theorem.
2.Find the points of inflection for the function $f(x) = x^4 - 8x^3 + 6$.

(b) Show that $f(x) = |x|$ is continuous but not differentiable at $x = 0$. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) Choose the correct option for the following questions (Each of 01 marks) **(03)**

1.Which of the following is an one-to-one function?
$(a) f(x) = x^2$ $(b) f(x) = x^3$
$(c) f(x) = \sin x$ $(d) f(x) = |x|$

2.If $f(x) = x + 5$ and $g(x) = x^2 - 3$ then, $(gof)(-2) =$________________.
$(a) 6$ $(b) -6$
$(c) x^2 + 10$ $(d) 4$

3.$\frac{d^n}{dx^n}[sin(ax + b)] =$____________________.
$(a) a^n sin\left(ax + b + \frac{n}{2}\pi\right)$ $(b) a^n sin\left(ax + \frac{n}{2}\pi\right)$
$(c) a^n cos\left(ax + b + \frac{n}{2}\pi\right)$ $(d) sin\left(ax + b + \frac{n}{2}\pi\right)$

(b) Separate the intervals in which function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing. **(03)**

(c) Find the root of the equation $e^{2x} = x + 6$,correct upto two decimal places using Newton Raphson's method. **(03)**

Q.3. A) Answer the following questions (Each of 04 marks) **(08)**

(a)Find the extreme values of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

(b)If $u = tan^{-1}(x^2 + 2y^2)$ then, show that
(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$
(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u . \cos 3u$.

Q.3. B) Answer the following questions (Any two)

(a) Do as directed (Each of 02 marks) **(04)**
1.Find the equation of the tangent plane to the surface $z + 8 = xe^y \cos z$ at the point (8,0,0).
2.Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = x - y, v = x + y$.

(b) Expand $sin\left(x + \frac{\pi}{4}\right)$ in powers of x.Hence, find the value of $\sin 44^o$. **(04)**

(c) Trace the curve $y^2(2a - x) = x^3,\ a > 0$. **(04)**

Q.4. A) Answer the following questions.

(a) Do as directed (Each of 02 marks) **(04)**

1.Verify Lagrange's mean value theorem for the function $f(x) = 1 - x^2$ where, $0 \leq x \leq 2$.

2.Expess $\frac{\partial w}{\partial r}$ in terms of r and s if $w = x + 2y + z^2, x = \frac{r}{s}, y = r^2 + \log s, z = 2r$.

(b) A soldier placed at a point (3,4) wants to shoot a fighter plane of an enemy which is flying along the curve $y = x^2 + 4$ when it is nearest to him. Find such distance. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Choose the correct option for the following questions (Each of 01 marks) **(03)**

1.The curve of $x^3 + y^3 = 3axy$ is symmetrical about

$(a) y = x$ $(b) x - axis$
$(c) y - axis$ $(d) none\ of\ these$

2. $\lim_{x\to 0} \frac{\sin x}{x} =$___________________.

$(a) 0$ $(b) -1$
$(c) 1$ $(d) 2$

3.$\frac{d}{dx}\left[x^{\frac{1}{2}}\right] =$___________________.

$(a) x^{\frac{5}{2}}$ $(b) \frac{3}{2}x^{\frac{5}{2}}$
$(c) \frac{1}{2}x^{-\frac{1}{2}}$ $(d) x^{-\frac{1}{2}}$

(b) Find the n^th^ derivative of $y = \frac{x}{1+3x+2x^2}$ **(03)**

(c) If $u = f(x^2 + 2yz, y^2 + 2zx)$, prove that **(03)**

$(y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0.$