Seat No: ______________ **Enrollment No: ____________________**

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech., Winter 2017 - 18 Examination

Semester: 1 **Date: 21/12/2017**

Subject Code: 03191101 **Time: 2:00 pm to 4:30 pm**

Subject Name: Mathematics-I **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Objective Type Questions : (All are compulsory) (Each of one mark) **(15)**

1. If $A = A^T$ then the Matrix is called
   (a) Symmetric (b) Skew-symmetric
   (c) Hermitian (d) Skew-Hermitian
2. The series $\sum_{n=1}^{\infty} \frac{1}{n^5}$ is
   (a) Convergent (b) Divergent
   (c) Oscillatory (d) None of these
3. The curve $a^2x^2 = y^2(2a - y)$ is symmetric about
   (a) x-axis (b) y-axis
   (c) x=y (d) None of these
4. If $z = xy\, f(x/y)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$
   (a) z (b) 0
   (c) 1/z (d) 2z
5. A matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is in
   (a) Row echelon form (b) Reduced row echelon form
   (c) Both (d) None
6. If $Z_1 = 1 + 2i$ & $Z_2 = 1 + 3i$ then $Z_1 + Z_2$ is ________.
7. The Eigen values of A = $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ is ___________.
8. If $r = -1$ in geometric series $a + ar + ar^2 + ... + ar^{n-1} + ...$ then series is ________.
9. If the tangents at a point are real and distinct, then the double point is called a_________.
10. For an implicit function f(x, y) = c, the value of $\frac{dy}{dx}$ is ___________.
11. Apply De- Moiver's theorem for $(\cos\theta + i\sin\theta)^n$ = __________.
12. The Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ *for* $u = x - y, v = x + y$ is__________.
13. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a __________ matrix.

14. Tell whether the series $\sum_{n=1}^{\infty} n \sin\frac{1}{n}$ is convergent or divergent?

15. Find $\frac{\partial f}{\partial z}$, for f(x,y,z)=$x^2yz^2$ at (1,1,1,).

Q.2 Answer the following questions. (Attempt any three) **(15)**

A). Solve following system of linear equations by Gauss Jordan method

$$x+2y+z=5;\ -x-y+z=2;\ y+3z=1.$$

B). If $u=\log(\tan x+\tan y+\tan z)$ then show that $\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}+\sin 2z\frac{\partial u}{\partial z}=2$.

C). Test the convergence of $\sum_{n=1}^{\infty}\frac{2n^2+2n}{5+n^5}$

D). Find the square roots of $1-\sqrt{3}i$

Q.3 **(A)** (1) Find the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(1,-3)$ if $f(x,y)=x^4+3x^2y+y^3-1$. **(07)**

(2) Find the limit, if it exists: $\lim_{(x,y)\to(0,0)}\frac{xy\cos y}{3x^2+y^2}$.

(B) (1) Test the convergence of the series $\sum_{n=0}^{\infty}\frac{n3^n(n+1)!}{2^n n!}$ **(08)**

(2) Prove that $\frac{(\cos 5\theta-i\sin 5\theta)^2(\cos 7\theta+i\sin 7\theta)^{-3}}{(\cos 4\theta-i\sin 4\theta)^9(\cos\theta+i\sin\theta)^5}=1$

OR

(B) (1) Trace the curve $y^2(2a-x)=x^3,\ a>0$. **(08)**

(2) Using slicing method find the volume of a solid ball of radius a.

Q.4 **(A)** (1) Find the area of the region between the X-axis and the graph of **(07)**

$$f(x)=x^3-x^2-2x,\ -1\le x\le 2.$$

(2) Find the Rank of the following matrix: $A=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}$

OR

(A) Find the Eigen values and Eigen vectors of the following matrix **(07)**

$$A=\begin{bmatrix}-2&2&3\\2&1&6\\3&6&6\end{bmatrix}$$

(B) (1) Expand f(x) = $x^4-11x^3+43x^2-60x+14$ using Maclaurin's series. **(08)**

(2) Use the Chain Rule to find $\frac{dw}{dt}$ for $w=xe^{y/z},\quad x=t^2,\quad y=1-t,\quad z=1+2t$