Seat No: _____ Enrollment No: _____

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech. Summer 2018 - 19 Examinations

Semester: 1 **Date: 17/05/2019**
Subject Code: 203191101 **Time: 02:00pm To 04:30pm**
Subject Name: Mathematics-I **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 **Objective Type Questions** (All are compulsory) (Each of one mark) **(15)**

1. The series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}-\frac{1}{2^5}+....\infty$ is _____
(a) convergent (b) divergent
(c) finitely oscillating (d) none of these

2. Cos(π - θ) = _____
(a) $Sin\theta$ (b) $-Cos\theta$
(c) $-Sin\theta$ (d) $Cos\theta$

3. $A=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ is _____
(a) Row echelon (b) Reduce row echelon
(c) both (d) none of these

4. Which type of integral $\int_{-1}^{1}\frac{1}{x^3}dx$ is _____
(a) Type –I (b) Type-II
(c) Type-III (d) none of these

5. Which of the following is an identity?
A. $\csc x=\sin x\cos x$ B. $\csc x=\cos x\cot x$
C. $\frac{1}{\csc x}=\cos x\cot x$ D. $\frac{1}{\csc x}=\cos x\tan x$

6. If $f(x,y)=x^2+2xy+y^2$ then find $\frac{\partial f}{\partial x}$ at (1,-2).

7. Write polar form of $z=1+i$.

8. Find the eigen value of matrix $A=\begin{bmatrix}3 & 2\\ 3 & 8\end{bmatrix}$

9. What is the value of $\Gamma\frac{1}{2}$?

10. If $f(x,y)=x^3+y^3-2xy^2$ then find f_{yx}

11. Find the Jacobian of $u=x^2-y^2$, $v=2xy$

12. What value of p, series $\sum_{n=0}^{\infty}\frac{1}{n^p}$ is convergent?

13. If $z=2+3i$ then find $z\bar{z}$.

14. If $A=\begin{bmatrix}3 & 0\\ 0 & 4\end{bmatrix}$ then find A^3.

15. State the Roll's theorem.

Q.2 **Answer the following questions. (Attempt any three)** **(15)**

A If $u = \log(\tan x + \tan y + \tan z)$ then prove that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$

B Find the Taylor series expansion of $f(x) = x^3 - 2x + 4$ about a=2

C Solve the following system by gauss- Elimination method
$-x + 3y + 4z = 30$; $3x + 2y - z = 9$; $2x - y + 2z = 10$

D Find the cube root of -8i

Q.3 A **(i)** Prove that $\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5} = 1$ **(04)**

(ii) Solve $\lim_{x \to \frac{\pi}{2}} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ **(03)**

B **(i)** If $u = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -2u$ **(04)**

(ii) Test the convergence of the series $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots$ **(04)**

OR

B **(i)** If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **(04)**

(ii) Test for convergence $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$ **(04)**

Q.4 A **(i)** Find eigen values and eigen vectors of the matrix. A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ **(04)**

(ii) Solve $\int_0^\infty e^{-\sqrt{x}} \sqrt[4]{x}\, dx$ **(03)**

OR

A **(i)** Using Caley-Hamilton theorem find inverse of $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ **(04)**

(ii) Find the area of the region between the x-axis the graph of **(03)**
$f(x) = x^3 - x^2 - 2x$; $-1 \le x \le 2$

B **(i)** Find the maxima and minima of the function $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **(04)**

(ii) Solve $\int_0^3 (3 - x)^{(\frac{1}{2})} x^{\frac{5}{2}} dx$ **(04)**