Seat No: _____________ Enrollment No: ________________

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech. Summer 2018 - 19 Examination

Semester: 1 **Date: 11/05/2019**
Subject Code: 03191101 **Time: 02:00 pm to 04:30 pm**
Subject Name: Mathematics-I **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Objective Type Questions : (All are compulsory) (Each of one mark) **(15)**

1. According to De-Movire's theorem, for any rational number n, $(\cos\theta + i\sin\theta)^n =$ ______.
   (a) $\cos n\theta + i\sin n\theta$ (b) $\cos n\theta$ (c) $\cos n\theta + i\sin\theta$ (d) $\cos n\theta - i\sin n\theta$
2. If $A = -A^T$, then the Matrix A is called
   (a) Symmetric (b) Skew-symmetric (c) Hermitian (d) Skew-Hermitian
3. Find the product of (4 + i) and (4 – i).
   (a) 15 (b) 17 (c) $\sqrt{17}$ (d) 16
4. The volume of a solid generated by revolving the curve $f(x, y) = 0$ about y-axis is given by
   (a) $\int_a^b \pi y^2 dx$ (b) $\int_a^b y^2 dx$ (c) $\int_a^b \pi x^2 dx$ (d) $\int_a^b x^2 dy$
5. Suppose that $f(x, y)$ and its first and second partial derivative are continuous throughout a disk centered at (a, b) and that $f_x(a, b) = f_y(a, b) = 0$. Then, for $r = \frac{\partial^2 f}{\partial x^2}$, $s = \frac{\partial^2 f}{\partial x \partial y}$, $t = \frac{\partial^2 f}{\partial y^2}$ f has a saddle point at (a, b) if
   (a) $rt - s^2 > 0$ (b) $rt - s^2 < 0$ (c) $rt - s^2 = 0$ (d) None of these
6. Let $\sum a_n$ be a series with $a_n \geq 0$ $for n \geq N$ and suppose that $\lim_{n\to\infty} |a_n|^{1/n} = L$ then the series converges if __________.
7. If the tangents are real and coincident, then the double point is called a _______.
8. The curve $y^2 = x$ is symmetric about x-axis. (True/ False)
9. If $u = y^2 e^{\frac{y}{x}} + x^2 \tan^{-1}\left(\frac{x}{y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$___________.
10. Find the rank of the matrix $\begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$.
11. Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for $u = x^2 - y^2, v = 2xy$.
12. If $ysinx = xcosy$, find $\frac{dy}{dx}$.
13. The Eigen values of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ is ___________.
14. Test the Convergence of the sequence $\{2^n\}$.
15. Test the convergence of series $\sum_{n=1}^{\infty} n \sin\frac{1}{n}$

Q.2 Answer the following questions: (Attempt any three) **(15)**

A) Solve following system of linear equations by Gauss Jordan method
$x + 2y + z = 5;\ -x - y + z = 2;\ y + 3z = 1$

B) Test the convergence of the series $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots$

C) If $u = \log(\tan x + \tan y + \tan z)$, then show that $\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$.

D) For $Z = 1 - i$, find the Arg (Z), arg (Z) and convert it in to polar form.

Q.3(A) Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ **(07)**

(B) (i) Find the limit, if it exists: $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$ **(04)**

(ii) Find the equation of the tangent plane and normal line to the surface $z + 8 = xe^y cosz$ at the point (8,0,0). **(04)**

OR

(B) (i) Using slicing method, find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{2x}$ from 0 to 1. **(04)**

(ii) Find the whole area bounded by the circle $r = 2a\ sin\theta$. **(04)**

Q.4(A) (i) Prove that $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$ *is divergent* **(03)**

(ii) Test the convergence of series $\sum_{n=1}^{\infty} \left(\frac{n}{2n+5}\right)^n$ **(04)**

OR

(A) Trace the curve $r = 2(1 + \cos\theta)$ **(07)**

(B) (i) Prove that $\frac{(\cos 5\theta - i\sin 5\theta)^2 (\cos 7\theta + i\sin 7\theta)^{-3}}{(\cos 4\theta - i\sin 4\theta)^9 (\cos\theta + i\sin\theta)^5} = 1$ **(04)**

(ii) If $f(x,y) = x^3 + y^3 - 2xy^2$, find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ of f at (1,-1) **(04)**