Seat No: _____________ Enrollment No: ______________

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech. Summer 2017 - 18 Examination

Semester: 1 **Date: 06/06/2018**
Subject Code: 03191101 **Time: 02:00 pm to 04:30 pm**
Subject Name: Mathematics-I **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Select correct alternative: (Each of one mark) **(15)**

1. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 4 & -6 & 10 \\ 7 & -2 & 16 \end{bmatrix}$ then trace of A is ___ .
(a) 0 (b) 9 (c) 11 (d) cannot be determined

2. If n is any rational number, then $(\cos\theta + i\,sin\theta)^n =$____ .
(a) $\cos n\theta + i sin\, n\theta$ (b) $\cos^n\theta + i\sin^n\theta$ (c) 1 (d) none of these

3. A complex number z is real if ___ .
(a) $Im(z) = 0$ (b) $Re(z) = 0$ (c) $Im(z) \neq 0$ (d) none of these

4. The area bounded by the x-axis and the curve $y = f(x)$ for $a \le x \le b$, is equal to ___ .
(a) $\int_a^b f(x)dx$ (b) $\int_a^b x\, f(x)dx$ (c) $\int_{-\infty}^{\infty} f(x)dx$ (d) none of these

5. A double point is called a node if the tangents to the curve at that point are _____ .
(a) real and coincident (b) real and distinct (c) imaginary (d) none of these

6. If $f(x,y) = x^2 - y$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

7. Find the rank of the matrix $\begin{bmatrix} -1 & 3 \\ 4 & 3 \end{bmatrix}$.

8. Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for $A = x - y, v = x + y$.

9. State whether the sequence $\{2^n\}$ is convergent or divergent.

10. Find $Arg(z)$ for $z = 1 + i$.

11. Reduce to the complex number $z = (1 - i)^4$ to a real number.

12. If $u_n = \left\{\frac{1}{1+\frac{1}{n}}\right\}$ then find $\lim_{n\to\infty} u_n$.

13. The matrix $\begin{bmatrix} 1 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is in row echelon form. (State whether true or false.)

14. State whether the given curve $x = y^2$ is symmetric about x-axis or not.

15. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ then what are the Eigen values of A^2?

Q.2 Answer the following questions. (Attempt any three) **(15)**

A) Trace the curve $y^2(2a - x) = x^3, a > 0$.

B) Simplify using De Moivre's theorem: $\frac{(cos2\theta + isin2\theta)^{\frac{2}{3}}(cos\theta - isin\theta)^2}{(cos3\theta - isin3\theta)^2(cos5\theta - isin5\theta)^{\frac{1}{3}}}$.

C) If $u = x^3y^2\sin^{-1}\left(\frac{y}{x}\right)$ then find (a) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ (b) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

D) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}$.

Q.3 **A)** Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. **(07)**

B) Using Lagrange's method of undetermined multipliers, find the maximum value of $f(x, y, z) = x^2y^3z^4$, subject to the condition $x + y + z = 5$. **(08)**

OR

B) (i) Find the limit if it exists: $\lim_{(x,y)\to(0,0)} \frac{xy\,cosy}{3x^2+y^2}$. **(05)**

(ii) Find the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(1, -3)$ if $f(x, y) = x^4 + 3x^2y + y^3 - 1$. **(03)**

Q.4 **A) (i)** The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume. **(05)**

(ii) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{3^{2n}}{4^{2n}}$. **(02)**

OR

A) Find the area of the region enclosed between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \le x \le 2$. **(07)**

B) Test the convergence of: (i) $\sum_{n=1}^{\infty} \frac{2n^2+2n}{5+n^2}$ and (ii) $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \cdots$. **(08)**