Seat No: _____ **Enrollment No: _____**

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

B.Sc./IMSC Summer 2017-18 Examination

Semester: 4 **Date: 15/05/2018**
Subject Code: 11106252 **Time: 10:30am to 1:00pm**
Subject Name: Introduction of Differential Equations **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions. **(08)**

(a) Show that for a homogeneous differential equation $Mdx + Ndy = 0$, $\frac{1}{Mx+Ny}$ is an integrating factor provided $Mx + Ny \neq 0$.

(b) Solve: $\frac{dy}{dx} = \frac{(x+2y-3)}{(2x+y-3)}$.

Q.1. B) Answer the following questions (Any two) **(08)**

(a) Solve following differential equations.(Each of 02 marks)

1. $(D^3 - 3D^2 + 3D - 1)y = 0$.
2. $p^2 - 5p + 6 = 0$.

(b) Find the Orthogonal Trajectories of the family of Rectangular hyperbolas $y = \frac{c}{x}$.

(c) Solve: $\frac{dy}{dx} + \frac{y}{x} = x^2y^6$

Q.2. A) Answer the following questions. **(08)**

(a) Solve following differential equations. (Each of 02 marks)

1. $(D^2 + 4D)y = e^{2x}$.
2. $\frac{\partial^2 z}{\partial x^2} = z$.

(b) Solve: $(x^3y^3 + x^2y^2 + xy + 1)ydx + (x^3y^3 - x^2y^2 - xy + 1)xdy = 0$.

Q.2. B) Answer the following questions (Any two) **(06)**

(a) $y \sin 2x \ dx - (y^2 + \cos^2 x)dy = 0$.

(b) Solve: $\left(\frac{y^2z}{x}\right)p + xzq = y^2$ by Lagrange's Method.

(c) Solve: $y = 2px + y^2p^3$

Q.3. A) Answer the following questions. (Each of 04 marks) **(08)**

(a) State and prove Necessary and sufficient condition for a first order first degree Ordinary Differential Equation to be exact.

(b) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of air. If the temperature of the air is $290\ K$ and the substance cools from $370\ K$ to $330\ K$ in 10 minutes, find when the temperature will be $295\ K$.

Q.3. B) Answer the following questions. (Any two) **(08)**

(a) Solve $(D^2 - 2D - 3)y = \sin 2x$ by using method of undetermined coefficients.

(b) Solve $(D^3 - D^2 - 6D)y = 1 + x^2$.

(c) Solve $(D^2 + a^2)y = \sin ax$.

Q.4. A) Do as directed. **(08)**

(a) Check whether the differential equation $(x^2 - ay)dx = (ax - y^2)dy$ is exact or not.

(b) Form a partial differential equation from the relation $2z = a(x + y) + b$.

(c) Convert the Bernoulli's differential equation $x\frac{dy}{dx} + y = y^2 \log x$ to a linear differential equation.

(d) Check whether x, x^2, x^3 are linearly independent or not by using Wronskian.

Q.4. B) Select the most appropriate answer for the following multiple choice questions. **(06)**

(1) Which of the following is general solution of the differential equation $xdy - ydx = 0$?

a) $xy = c$ b) $\frac{y}{x} = c$

c) $x + y = c$ d) $x^2 + y^2 = c$

(2) Which of the following is the general solution of the differential equation $(y - px)(p - 1) = p$?

a) $y = cx + \frac{c}{c-1}$ b) $y = -cx + \frac{c}{c+1}$

c) $y = c_1x + \frac{c_2}{c_2-1}$ d) $y = -c_1x + \frac{c_2}{c_2+1}$

(3) Which of the following is a non-linear differential equation?

a) $\frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} = y$ b) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$

c) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$ d) $\frac{d^2y}{dx^2} + \sin y \frac{dy}{dx} = y$

(4) What is the order and degree of the partial differential equation $\frac{\partial z}{\partial x} - \frac{\partial^2 z}{\partial y^2} = z$?

a) order-1, degree-2 b) order-2, degree-1

c) order-1, degree-1 d) order-2, degree-2

(5) $\frac{1}{D^2+9} \sin 2x =$_____.

a) $\frac{\sin 2x}{13}$ b) $\frac{\sin 2x}{5}$

c) $-\frac{\sin 2x}{4}$ d) $\frac{\sin 2x}{7}$

(6) $\frac{1}{D^2-1} x^3 =$_____.

a) $6x - x^3$ b) $3x^2 - x^3$

c) $6x^2 - x$ d) $3x$