

PARUL UNIVERSITY
FACULTY OF APPLIED SCIENCE
B.Sc., Summer 2017-18 Examination

Semester: 3
Subject Code: 11106202
Subject Name: Laplace Transform and Fourier Series

Date: 24/05/2018
Time: 10:30am to 1:00pm
Total Marks: 60

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions (08)

- (a) Find $L[\cos\sqrt{t}/\sqrt{t}]$
- (b) Let $L[f(t)] = F(s)$ then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty f(x)dx$ provided that $\lim_{n \rightarrow \infty} \frac{f(t)}{t}$ exists

Q.1. B) Answer the following questions (Any two)

- (a) Find Laplace Transform of $e^{-4t} \int_0^t t \sin 3t dt$ (04)
- (b) State and Prove Existence Theorem of Laplace Transform (04)
- (c) Find the Laplace Transform of $(1+2t-3t^2 + 4t^3) u(t-2)$ and hence evaluate $\int_0^\infty e^{-t}(1+2t-3t^2 + 4t^3) u(t-2) dt$. (04)

Q.2. A) Answer the following questions.

- (a) Find Laplace Transform of $f(t)$, where $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$ (04)
- (b) State and Prove Second Shifting theorem for Laplace Transform (04)

Q.2. B) Answer the following questions (Any two)

- (a) Find $L[t^2 \cosh \pi t]$ (03)
- (b) Find $L[f(t)]$ and $L[f'(t)]$ for $f(t) = \begin{cases} 3 & 0 \leq t < 5 \\ 0 & t > 5 \end{cases}$ (03)
- (c) Find Laplace Transform of $\int_0^t \int_0^t \sin ax dx dx$ (03)

Q.3. A) Answer the following questions (08)

- (a) Find the Inverse Laplace transform using Partial Expansion $\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}$
- (b) Find the Fourier Series of $f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$

Q.3. B) Answer the following questions (Any two)

- (a) Find the Fourier Series expansion of $f(x) = e^{-|x|}, -\pi < x < \pi$ (04)
- (b) Using Laplace transform, Prove that $\int_0^\infty \frac{\sin(t)}{t} dt = \frac{\pi}{2}$ (04)
- (c) Define: Convolution of Two Functions. Prove that Convolution of Two functions is Commutative. (04)

Q.4. A) Answer the following questions.

- (a) Answer the following questions (Each of 02 marks) (04)
1. Find the Fundamental period for the following functions
 - I. $\sin \pi + \cos 2\pi$
 - II. $\sin \pi + \cos \pi/2$
 2. Show that $f(x) = x^2$ and $f(x) = x^3$ are orthogonal functions on the interval $[-1, 1]$
- (b) Find the Half range cosine series of $f(x) = (x-1)^2$ in $0 < x < 1$. (04)

Q.4. B) Answer the following questions (Any two)

- (a) Find $t * e^t$ (03)
- (b) If $L^{-1}\left[\frac{s}{(s^2+1)^2}\right] = \frac{1}{2} t \sin t$, find $L^{-1}\left[\frac{8s}{(4s^2+1)^2}\right]$ (03)
- (c) $L^{-1}[\tan^{-1} s]$ (03)