Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

B.Sc. Winter2017-18 Examination

Semester: 1 **Date: 28-12-2017**
Subject Code: 11106101 **Time: 10:30AM to 01:00 PM**
Subject Name: Mathematics-I **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions (Each of 04 marks) **(08)**

(a) State and prove necessary condition for extreme values.

(b) Verify the Lagrange's mean value theorem for $f(x)=\log x,\ x\in[1,e]$

Q.1. B) Answer the following questions (Any two)

(a) 1. Find first order derivative of $y=\left(x+\dfrac{1}{x}\right)\left(x-\dfrac{1}{x}+1\right)$ **(04)**

2. State the sandwich theorem. Using theorem find $\lim\limits_{x\to 0} f(x)$ where

$$1-\frac{x^2}{2}\le f(x)\le 1+\frac{x^2}{2}$$

(b) Find $\dfrac{d^n y}{dx^n}$ for $y=e^{ax}\cos^2 bx$ **(04)**

(c) If $u=\log(\tan x+\tan y+\tan z)$ then prove that

$$\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}+\sin 2z\frac{\partial u}{\partial z}=2$$ **(04)**

Q.2. A) Answer the following questions.

(a) 1. If $y\log(\cos x)=x\log(\sin y)$ then find $\dfrac{dy}{dx}$. **(04)**

2. If $u=2xy, v=x^2-y^2\ and\ x=r\cos\theta, y=r\sin\theta$ then evaluate $\dfrac{\partial(u,v)}{\partial(r,\theta)}$.

(b) Prove that $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{vu'-uv'}{v^2}$. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) 1. Define one-one function. **(03)**
2. Define Increasing function.
3. Define function.

(b) Check whether the following function is invertible or not for $y=1+x^2, x\in[-2,0]$. If yes then find domain of inverse function. **(03)**

(c) $f(x)=3|x|+4|x-1|,\ x\in R$ has minimum value 3 at $x=1$ **(03)**

Q.3. A) Answer the following questions: **(08)**

(a) State and prove Rolle's theorem.

(b) Discuss the continuity of $f(x)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2} & ,(x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$ at origin.

Q.3. B) Answer the following questions (Any two)

(a) 1. Find the value of $(f \circ g)'$ at the given value of x for **(04)**

$$f(x) = 1 - \frac{1}{u}, u = g(x) = \frac{1}{1-x}, x = -1$$

2. Find $\frac{dy}{dx}$ *for* $x = 2\cos t - \cos 2t, y = 2\sin t - \sin 2t.$

(b) Trace the curve $r = 1 + \cos\theta$ **(04)**

(c) Using Taylor's formula expand $\tan^{-1}\left(\frac{y}{x}\right)$ at the point (1,1) upto second degree term. **(04)**

Q.4. A) Answer the following questions.

(a) 1. Find the equations of tangent plane to the surface $2x^2 + y^2 + 2z = 3$ at the point (2,1,-3) **(04)**

2. Find the asymptotes of the curve $y^2 = \frac{x(x-a)(x-2a)}{(x+3a)}$.

(b) State and prove Euler's theorem on homogeneous functions. **(04)**

Q.4. B) Answer the following questions (Any two)

(a) 1. Define Change of sign **(03)**

2. Define Even function.

3. Define Limit for one variable.

(b) If $u = \frac{1}{x^2} + \frac{1}{y^2} + \frac{\log x - \log y}{x^2 + y^2}$ then prove that **(03)**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 6u$$

(c) Use chain rule to find $\frac{dz}{dt}$. If $z = x^2 e^y, x = \sin t, y = t^3$. **(03)**