PARUL UNIVERSITY
FACULTY OF ENGINEERING & TECHNOLOGY
B.Tech. Winter 2022 - 23 Examination

Semester: 1   **Date: 28/01/2023**
Subject Code: 303191101   **Time: 02:00 pm to 04:30 pm**
Subject Name: Mathematics-I   **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Objective Type Questions.   **(15)**

1. $\frac{\partial f}{\partial x}$ for $f = x^y$ is
   (a) yx^{y-1} (b) xy^{x-1} (c) x^{y-1} (d) None of the above

2. An eigenvalue of 2×2 matrix A is 3, the eigenvalue of $A^2 + 3I$ is _____
   (a) 9   (b) 6   (c) 3   (d) 12

3. Which of the following is reduced row echelon form?
   (a) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

4. The value of $a = $ _____, so that the system
$$(a - 1)x + 2y = 0$$
$$3x + (a - 2)y = 0 \quad \text{has non trivial solution.}$$
   (a) 4   (b) 1   (c) 0   (d) -4

5. The degree of homogeneous function $f(x,y) = \frac{x^2+y^2}{x+y}$
   (a) 0   (b) 1   (c) 2   (d) 3

6. The degree and order of the differential equation
$\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^2 + 2y = 3$ is _____ and _____ respectively.
   (a) 2,3   (b) 3,2   (c) 1,2   (d) 2,1

7. Which of the following series is convergent?
   (a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (d) $\sum_{n=0}^{\infty} \cos(n\pi)$

8. $\Gamma\left(\frac{1}{2}\right) = $ _____.

9. Write the relation between Gamma and Beta function.

10. Every square matrix satisfies its own characteristic equation. **(True/False)**

11. If $f(x) = x\sin x$ in $(-\pi, \pi)$ then the value of $b_n = $ _____

12. Find the normal line to the sphere $x^2 + y^2 + z^2 = 3$ at the point $(1,1,1)$.

13. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ then eigenvalues of matrix A are _____ and _____.

14. $\lim_{(x,y)\to(1,1)} \frac{x-y}{\sqrt{x}-\sqrt{y}} = $ _____

15. Express the quadratic form $Q(x,y) = x^2 + 3y^2 + 2xy$ in the matrix form.

Q.2 Answer the following questions. (Attempt any three)   **(15)**

A) Solve the following system of equations by using Gauss elimination method
$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

B) Discuss the continuity of

$$f(x,y) = \begin{cases} \frac{x^2-y^2}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$ at origin.

C) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.

D) Find the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at point $(1,2)$ if $f(x,y) = x^2 + 2xy + 3y^2 - 1$

Q.3 A) (i) Solve $\frac{dy}{dx} + y\tan x = \sin 2x$ **(04)**

(ii) Evaluate $\int_0^\infty \frac{1}{1+x^2} dx$ **(03)**

B) Find the non-singular matrix P that diagonalizes matrix A and determine $P^{-1}AP$ where **(08)**
$A = \begin{bmatrix} 5 & 7 \\ -2 & -4 \end{bmatrix}$.

OR

B) Find the Fourier Series of $f(x) = x^2$ in the interval $(-\pi, \pi)$. **(08)**

Q.4 A) If $u = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ prove that **(07)**

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin 2u}{2}$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u$

OR

A) (i) Check whether the differential equation is exact or not and hence, solve **(04)**
$$2xydx + (1+x^2)dy = 0$$

(ii)Test the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+2n}{5+n^5}$ **(03)**

B) Discuss the maxima and minima of the function **(08)**
$f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$