Seat No:______________ Enrollment No:______________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc., Summer 2022-23 Examination

Semester: 4 **Date: 29-03-2023**
Subject Code: 11206259 **Time: 2:00pm to 4:30pm**
Subject Name: Dynamical Systems and Control **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions. (Any one) **(08)**

(a) State and prove Kalman's condition for controllability.

(b) Let $A(t)$ be a continuous $n \times n$ matrix defined on a closed and bounded on interval I, then IVP $\dot{x}(t) = A(t)x(t), x(t_0) = x_0, t_0 \in I$, has a unique solution on I.

Q.1. B) Answer the following questions (Any two)

(a) Solve, $y(0) = 1, \frac{dy}{dx} = (xy + 1)$ using Picard's method till 3rd iterations. **(04)**

(b) Check the following system is controllable or not? **(04)**

$$\dot{x}_1 = -\alpha x_1$$
$$\dot{x}_2 = \alpha x_1 - \beta x_2 + u$$

(c) Solve the following IVP, **(04)**

$$\frac{dX}{dt} = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix} X, X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

where, $X = [x_1, x_2\ ]^T$

Q.2. A) Answer the following questions.

(a) Find the fundamental matrix of for, **(04)**

$$\begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

(b) Linearize the following system, **(04)**

$$\dot{x}(t) = 0.1\, x - 0.005xy$$
$$\dot{y}(t) = -0.4\, y + 0.008xy$$

$x(0) = 130, y(0) = 40.$

Q.2. B) Answer the following questions (Any two)

(a) Convert the following equation into the system of differential equation. **(03)**

$$\frac{d^3x}{dt^3} - 6\frac{d^2x}{dt^2} + 11\frac{dx}{dt} - 6x = 0$$

(b) Discuss the stability of the differential equation, **(03)**

$$\dot{x}(t) = -x, x(t_0) = x_0$$

(c) Solve the following system also show that the system $\dot{x} = y, \dot{y} = -x$ is stable. **(03)**

Q.3. A) State and prove Grownwall's inequality. **(08)**

Q.3. B) Answer the following questions.

(a) Consider the IVP $\dot{x} = x^2, x(0) = 2$, find the value of h, for the following domain, **(04)**

$$R = \{(t, x): |t| \leq 2, |x - 1| \leq 2\}$$

(b) Determine the nature of following equation regarding positive or negative definiteness, **(04)**

$$x_1^2 + 2x_1x_2 + 2x_2^2 = 0$$

Q.4. A) Answer the following questions.

(a) Sketch the trajectories for the system, $\dot{X} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} X$ **(04)**

(b) Solve, $\frac{dy}{dx} + ysinx = sinx$ **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Define the term Observability in dynamical system. **(03)**

(b) Write statement of Poincare- Bendixon theorem. **(03)**

(c) Construct the difference equation for the following statement, **(03)**

Consider moose population of $m_0 = 5000$ and grow by 5% per year.