Seat No: _____________ Enrollment No: ___________________

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech. Winter 2019 - 20 Examination

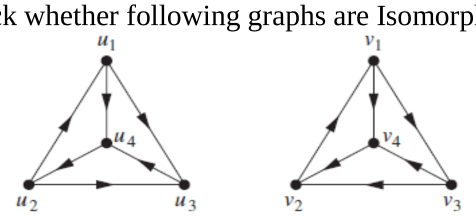
Semester: 3 **Date: 04/12/2019**
Subject Code: 203191202/03191202 **Time: 2:00pm to 4:30pm**
Subject Name: Discrete Mathematics **Total Marks: 60**

Instructions:

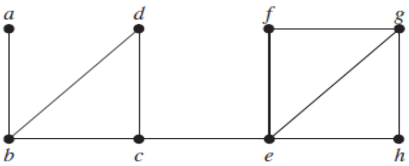
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Objective Type Questions: (All are compulsory) (Each of one mark) **(15)**

1. A simple path in a graph G that passes through every vertex exactly once is called a______.
2. A commutative ring which has no zero divisor is called ________.
3. "q $whenever$ p" is a Biconditional statement.(True/False)
4. Write Rule of Inference for Modus Tollens and Modus Ponens.
5. If $A = \{1,2,3,4,5\}$ & the set $B = \{1,2,3\}$ then write the relation from A to B when $(a, b) \in R$ if and only if "a is a $divisor$ of b".
6. State: Pigeonhole Principle
7. Translate the logical equivalence $(\mathrm{T} \wedge \mathrm{T}) \vee \neg\mathrm{F} \equiv \mathrm{T}$ into an identity in Boolean algebra.
8. Suppose that $A = \{1,2,3\}$ and $B = \{1,2\}$. Let R be the relation from A to B containing (a, b) if $a \in A, b \in B, and\ a > b$. What is the matrix representing R?
9. A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?
10. Check whether following graphs are Isomorphic?



11. Find the cut edges in the graph G shown in Figure.



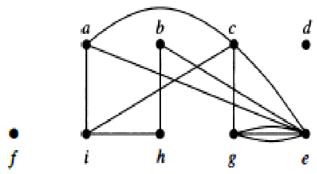
12. Which of the following operator is not an associative operator on the set of real numbers?
    (a) Usual addition (b) Usual subtraction (c) Usual multiplication (d) none of them
13. A relation R on a set A is said to be equivalence relation, if R is ______.
    (a) reflexive, transitive and symmetric (b) anti-symmetric, transitive and symmetric
    (c) anti-symmetric, reflexive, symmetric (d) reflexive, transitive and anti-symmetric
14. A positive integer p greater than 1 is called ______if the only positive factors of p are 1and p.
    (a) Composite (b) Prime (c) Rational (d) None of these
15. Which of following sentence are not propositions?
    (a) This problem is hard to be solved. (b) It is not a valid question
    (c) It is solvable only if x=2 (d) Will you be able to solve it?

Q.2 Do as directed: (Attempt any three) **(15)**

A) Prove by constructing truth table
   (i) $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.
   (ii) Show that $\neg(p \vee q) \equiv \neg p \wedge \neg q$ are logically equivalent.

B) Find the product-of-sums expansion for the function $F(x, y, z) = (x + y)\bar{z}$ using the table of values.

C) Find the number of vertices, number of edges, and degree of each vertex in the following Undirected graphs. Identify all isolated and pendant vertices. Verify Handshaking Theorem.



D) Let R and S be relations on a set A represented by the matrices $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find the matrices representing the following relations.

(a) $R \cup S$ (b) $R \cap S$ (c) $S \circ R$ (d) $R \circ S$ (e) $R \oplus S$

Q.3 A) (i) Draw the directed graph for the relation and check whether it satisfies Reflexive, Anti-symmetric Relation. $R = \{(a,b)/a \neq b; a,b \in A, A = \{1,2,3,4\}\}$ **(03)**

(ii) In each of the following cases, prove that $\mathbb{Z}$ under the operation $*$ is not an Abelian group.(a) $a * b = \frac{a}{b}$ (b) $a * b = |a + b|$ **(04)**

B) (a) Let *C(x)* be the statement "*x* has a cat," let *D(x)* be the statement "*x* has a dog," and let *F(x)* be the statement "*x* has a ferret." Express each of these statements in terms of *C(x)*, *D(x)*, *F(x)*, quantifiers, and logical connectives. **(02)**

Let the domain consist of all students in your class.

i) All students in your class have a cat, a dog, or a ferret.

ii) Some student in your class has a cat and a ferret, but not a dog.

(b) Write the converse, contrapositive of "If it snow tonight, then I will stay at home". **(02)**

(c) Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$. **(04)**

OR

B) (a) Prove that $(\mathbb{Z}_5^*, \cdot_5)$ is an abelian group. **(04)**

(b) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 1 & 4 \end{pmatrix} \in S_5$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix} \in S_5$ then Check whether σ and τ are (i) commutative. (ii) even or odd. **(04)**

Q.4 A) Prove that $R = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \text{ are real numbers} \right\}$ with usual matrix addition is an abelian group. Let the matrix multiplication be distributive over addition to make it a ring. Find the unit element of this ring. Is it a commutative ring? **(07)**

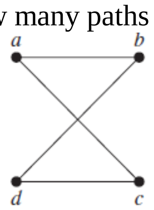
OR

A) (i) Use Euclidean algorithm to find the HCF of 4052 and 12576. **(03)**

(ii) A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these people majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business? **(02)**

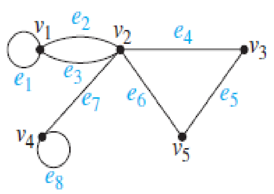
(iii) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department? **(02)**

B) (a) How many paths of length four are there from a to d in the simple graph G in Figure? **(04)**



(02)

(b) Represent the graph shown in the figure with an incidence matrix



(c) Draw the graphs: $K_{3,4}$, W_7

(02)