

**PARUL UNIVERSITY**  
**FACULTY OF ENGINEERING & TECHNOLOGY**  
**B.TECH MIDSEM EXAMINATION**  
**3<sup>th</sup> SEMESTER**  
**ACY-2022-23 (ODD SEM)**

**Subject Name (Code):** Complex Variables and PDE(203191205)

**Branch:** IC/CHEMICAL/MECHATRONICS/AERO/PETRO/BIO

**Date:** 09/08/2022

**Time:** 2:30 – 4:00

**Total Marks:** 40

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**Q.1 (A) One-line Questions**

05

- (1) Multiplicative Identity for entire complex number is \_\_\_\_\_
- (2) If  $Z_1 = x_1 + iy_1$  and  $Z_2 = x_2 + iy_2$  then find  $Z_1 + Z_2$
- (3) Argument of  $z = -i$  is \_\_\_\_\_
- (4) PDE  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  is known as \_\_\_\_\_
- (5) The order and degree of the PDE  $x \left(\frac{\partial^3 z}{\partial x^3}\right)^2 + y \left(\frac{\partial z}{\partial y}\right)^5 = z$  is,

**(B) Compulsory Questions**

05

- (1) Find modulus value of  $(2 - 5i)$
- (2) Define Analytic function.
- (3) Principal argument of  $Z = 1+i$
- (4) If  $f(z) = |z|^2$  then find  $\text{Re}\{f(z)\}$
- (5) A transformation of the type  $w = Az+C$  is called \_\_\_\_\_

**Q.2 Attempt any four (Short Questions)**

12

- (1) Form PDE from the relation  $z^2 = ax^2 + by^2$
- (2) By using the definition show that  $f(z) = iz^3$  is differentiable for all  $z$ .

(3) Determine whether the function  $f(z)$  is continuous at  $z = -i$

$$f(z) = \begin{cases} \frac{(z^2+3iz-2)}{z+i} & z \neq -i \\ 5 & z = -i \end{cases}$$

can the function be redefined to make it continuous at  $z = -i$ ?

(4) Evaluate  $\int_0^{2+i} z^2 dz$  along the line  $y = \frac{x}{2}$

(5) Evaluate  $\int_c \frac{e^z dz}{(z-1)(z-2)}$  where  $c$  is  $|z| = \frac{1}{2}$

**Q.3 Attempt any two**

08

(1) Form PDE from  $f(x + y + z, x^2 + y^2 + z^2) = 0$

(2) Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1) dz$  along the path  $x = t+1, y = 2t^2 - 1$

(3) Find the image of infinite strip  $\frac{1}{4} \leq y \leq \frac{1}{2}$  under the

transformation  $w = \frac{1}{z}$  Also show the regions graphically.

**Q.4** (A) Find the bilinear transformation which maps the points  $z = -1, 0, 1$  into the points  $w = -i, 1, i$

(B) Find root of  $z^3 = 2 + 2i$

OR

(B) Determine harmonic conjugate function of the given function  $v(x,y) = x^2 - y^2$  and also find analytic function  $f(z)$ .