Seat No: _____________ Enrollment No: ___________________

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech. Summer 2018 – 19 Examination

Semester: 3 **Date: 30/05/2019**
Subject Code: 03191201 **Time: 02:00 pm to 04:30 pm**
Subject Name: Mathematics 3 **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 A) Choose the correct answer (Each question carries ONE mark) **(15)**

1. If $y_1 = x, y_2 = 1$; The Wronskian $W\left(y_1, y_2\right)$ is
   (a) -1 (b) 1 (c) x (d) 0
2. An example for a function which neither even nor odd
   (a) x (b) sinx (c) cosx (d)none of these
3. Inverse Laplace Transform of $\frac{2!}{(s-2)^2}$ is
   (a) $2te^{2t}$ (b) t^2e^{-2t} (c) t^2e^{2t} (d) none of these
4. Laplace Transform of $\sinh t$ is
   (a) $\dfrac{1}{s^2+1}$ (b) $\dfrac{1}{s^2-1}$ (c) $\dfrac{s}{s^2+1}$ (d) $\dfrac{s}{s^2-1}$
5. The partial differential equation $5\dfrac{\partial^2 z}{\partial x^2} + 6\dfrac{\partial^2 z}{\partial y^2} = xy$ is classified as
   (a) elliptic (b) parabolic (c) hyperbolic (d) none of the above

B) Fill in blanks`(Each question carries ONE mark)

1. The general solution of $y'' - y' = 0$ is _______.
2. Solution of P.D.E. $pq = 1$ is ________.
3. Inverse Laplace transform of $F(s) = e^{-as}$ is ________.
4. If the Fourier Integral $f(x) = \int_0^\infty \left(A(w)\cos wx + B(w)\sin wx\right)dw$, formula of $B(w)$ is ____.
5. For the periodic function $f(x) = k, -\pi \le x \le \pi$, $b_n =$________.
6. $Z[1] =$ _________.

C) True or False (Each question carries ONE mark)

1. Lagrange's linear equation is of the form $Pp + Qq = R$
2. General solution of the homogeneous differential equation with constant coefficients whose roots are $m = 1 \; and \; 2$ is $y_c = c_1e^x + c_2e^{2x}$.
3. Inverse Laplace Transform of $F(s) = e^{-as}$ is $u(t-a)$.
4. Even function is symmetric about Origin.

Q.2 Answer the following questions. (Attempt any three) **(15)**

A) Using method of Variation of Parameter, Solve $y'' + y = \tan x$

B) Find the Fourier series of $f(x) = \pi x$ in the interval$(-\pi, \pi)$.

C) Using Laplace Transformation, solve the initial value problem

$$y'' + y = \sin 2t\,;\; y(0) = 2; y'(0) = 1$$

D) Solve the Cauchy- Euler differential equation $x^2y'' + 3xy' + y = 6$

Q.3 A) (I) Solve $y'''-y''+100y'-100y=0$ **(03)**

(II) Find the half-range Cosine Series of $f(x) = x^2$ in the interval $(0,\pi)$ **(04)**

B) (I) Find the Fourier Sine Integral of $f(x) = e^{-bx}$ **(04)**

(II) Find the Fourier Series of $f(x) = \frac{1}{2}(\pi - x)$ in the interval $(0,2\pi)$ **(04)**

OR

B) (I) Find $L^{-1}\left(\frac{2s+3}{s^2+9}\right)$. **(04)**

(II) Evaluate $L\left(t^2 \sin 2t\right)$ **(04)**

Q.4 A) Solve the differential equation using Z transform **(07)**

$$u_{n+1} + u_n = 1, \qquad u_0 = 0$$

OR

A) Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ where, the boundary condition are u (0,t) = u (L, t) = 0 t > 0 and the initial condition is u (x,0) = x, L being the length (0 < x < L). **(07)**

B) (I) Solve $(D^2 + 10DD' + 25D'^2) = e^{3x+2y}$ **(04)**

(II) Solve $p + q = sinx + siny$ **(04)**