Seat No:_____________ Enrollment No:_____________

PARUL UNIVERSITY

FACULTY OF APPLIED SCIENCE

M.Sc. Winter 2019-20 Examination

Semester: 1 **Date: 29/11/2019**
Subject Code: 11206104 **Time: 10:30am to 01:00pm**
Subject Name: TOPOLOGY **Total Marks: 60**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Answer the following questions. **(08)**

(a) Define topological space. Prove that an infinite set forms a topological space with the co-finite topology.

(b)Let X, Y be topological spaces and $\mathcal{B}$ be a basis for the topology on Y. A function $f: X \to Y$ is continuous if and only if $f^{-1}(B)$ is open for every $B \in \mathcal{B}$.

Q.1. B) Answer the following questions (Any two)

(a) Answer the following questions. (Each of 02 marks) **(04)**

1. Show that $\{[a,b] | a < b, a, b, \in \mathbb{R}\}$ will not form base for any topology $\mathcal{T}$ on $\mathbb{R}$.
2. Define Local Connectedness and Local Path-connectedness.

(b)Define regular and normal space. Prove that every normal space is regular. **(04)**

(c) Let $(X, \mathcal{T})$ be a topological space and $(Y, \mathcal{T}_Y)$ be its subspace. Then prove that$Y$ is compact in Y if and only if Y is compact in X. **(04)**

Q.2. A) Answer the following questions.

(a) Answer the following questions. (Each of 02 marks) **(04)**

1. Give an example of a separable space which is not second countable.
2. Prove that $f: R \to R$ be defined by $f(x) = 2x + 1$ is a homeomorphism.

(b)Prove that every metrizable space is a first countable space. **(04)**

Q.2. B) Answer the following questions (Any two)

(a) Answer the following.(Each of 01 marks) **(03)**

1. "Every metric space can also be seen as a topological space."$[True/False]$
2. "The product of two path connected spaces is necessarily connected" $[True/False]$
3. "Let $X = \{a, b, c\}$ then $\mathcal{T} = \{\emptyset, \{a\}, \{b\}, X\}$ is a topology on X." $[True/False]$

(b) Define limit point compactness. **(03)**

Prove that the subspace$A = \{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{Z}_+\right\}$ is limit point compact in $\mathbb{R}$.

(c) Prove that every second countable space is a separable space. **(03)**

Q.3. A) Answer the following questions. **(08)**

(a) Let (X, τ_1) and (Y, τ_2) be two topological spaces.
Show that$B = \{G_1 \times G_2 | G_1 \in \tau_1, G_2 \in \tau_2\}$is a base for some topology on $X \times Y$.

(b)Let X, Y be topological spaces and $f: X \to Y$ be a function, then the following are equivalent.
(i) f is continuous.
(ii) For each $x \in X$, and for each neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset V$.

Q.3. B) Answer the following questions (Any two)

(a) Answer the following. (Each of 02 marks) **(04)**

1. State Urysohn's Lemma.
2. Show that union of two compact sets is compact.

(b) A continuous image of a connected space is connected. **(04)**

(c) The components of X are connected disjoint subspaces of X whose union is X, each non-empty connected subspace of X intersects only one of them. **(04)**

Q.4. A) Answer the following questions.

(a) Answer the following.(Each of 02 marks) **(04)**

1. List All possible topologies on $\{a, b\}$.
2. Prove that a subspace of T_1 space is T_1.

(b)For any set A in topological space, $\bar{A} = A \cup A'$ **(04)**

Q.4. B) Answer the following questions (Any two)

(a) Answer the following.(Each of 01 marks) **(03)**

1. Define Path Components.
2. Define Housdorff Space.
3. Define Compact space.

(b) Prove that path connectedness is an equivalence relation on a topological space X. **(03)**

(c) A topological space X is T_1 if and only if every singleton subset of X is closed in X. **(03)**