

PARUL UNIVERSITY
FACULTY OF APPLIED SCIENCE
M.Sc./ IMSc, Winter 2019-20 Examination

Semester: 1/7

Subject Code: 11206103

Subject Name: Advanced Numerical Analysis

Date: 27-11-2019

Time: 10:30 am to 01:00 pm

Total Marks: 60

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1. A) Find the eigen values of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, using the power method up to five iteration. (08)

Q.1. B) Using House-holder's method, reduce the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ to the tri diagonal form. (08)

OR

Q.1. B) Apply Runge-Kutta 4th order method to find approximate value of y at $x = 0.2$ given that (08)
 $\frac{dy}{dx} = x + y^2$; $y(0) = 1$, $h = 0.1$

Q.2. A) Solve the Poisson equation $u_{xx} + u_{yy} = -81xy$, $0 < x < 1$, $0 < y < 1$ given that (08)
 $u(0, y) = 0$, $u(x, 0) = 0$, $u(1, y) = 100$, $u(x, 1) = 100$, $h = \frac{1}{3}$.

Q.2. B) Answer the following questions (Any two)

(a) Solve $y' = x^2 + y^2$; $y(0) = 1$ by Taylor's series method. Hence find the values of y at $x = 0.1$ (03)

(b) Using Euler's method, find an approximate value of y corresponding to $x = 0.4$, given that $y' = x + y$; $y(0) = 1$; $h = 0.2$ (03)

(c) Find the eigen values of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (03)

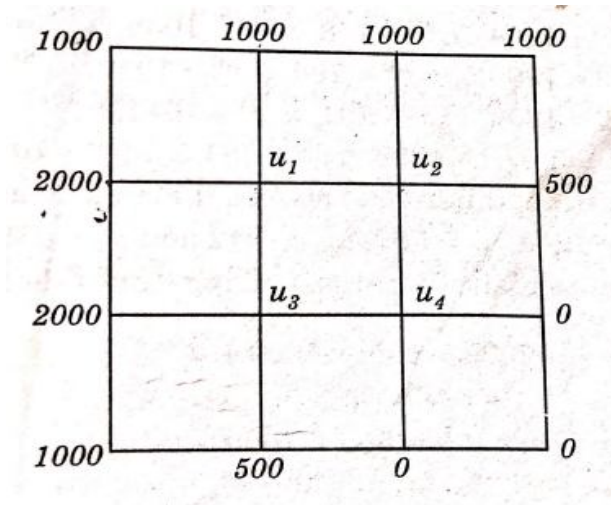
Q.3. A) Apply Milne's Predictor Corrector method to solve $y' = x - y^2$ in the range $0 \leq x \leq 1$ at $y(0) = 0$. (08)

Q.3. B) Using Jacobi's method, find all eigen values and the eigen vectors of the matrix (08)
 $\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$

OR

Q.3. B) Given that $\frac{dy}{dx} = x + y^2$; $y(0) = 1$ find an approximate value of y at $x = 0.5$, by modified Euler's method. (08)

- Q.4. A)** Given the values of $u(x, y)$ on the boundary of the square in the below figure, evaluate the function $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of this figure by Gauss Seidel method (08)



- Q.4. B)** Find the eigen values and eigen vector using inverse power method of the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ (06)

OR

- Q.4. B)** Apply Runge-Kutta 2nd order method to find approximate value of y at $x = 1.2$ given that (06)
- $$\frac{dy}{dx} = 3x + y ; y(1) = 1.3 , h = 0.1$$